Errata for *Tables of Integrals, Series, and Products* (8^{th} edition)

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NOTES

- The home page for this book is `http://www.mathtable.com/gr`
- The latest errata are available from `http://www.mathtable.com/errata/`
- The author can be reached at `ZwillingerBooks@gmail.com`
- This edition of the errata includes all the corrections in the paper: Dirk Veestraeten, *Some remarks, generalizations and misprints in the integrals in Gradshteyn and Ryzhik*, SCIENTIA, Series A: Mathematical Sciences, Vol. 26 (2015), pages 115–131.
- This document contains new material (starting below) following by corrections to the 8th edition (starting on page 16).
- The updates since the last set of errata (on April 2021) are shown with the date in the margin, as this line has. *7/2022*

NEW MATERIAL (TO ADD TO THE 9th EDITION)

1. On page XXX, add section XYZ Quadruple Integrals

   Where should this new section go?

   (Thanks to Robert Reynolds for suggesting the inclusion of the evaluations in this section.)

XYZ.1

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty (t+z)^{-m}(x+y)^{m-1} e^{-p(x+z)-q(t+y)} \log^k \left(\frac{a(x+y)}{t+z}\right) \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t$$
$$= \frac{1}{(p-q)^2}(2\pi i)^{k+1} e^{im\pi} p^{-m-1} q^{-m-1} \Big(-p^m q^m \Phi(e^{2m\pi i}, -k, \frac{\pi - i\log a}{2\pi})$$
$$+ qp^{2m}\Phi(e^{2m\pi i}, -k, \frac{\pi - i\log a - i\log p + i\log q}{2\pi}) + qp^{2m}\Phi(e^{2m\pi i}, -k, \frac{\pi - i\log a + i\log p - i\log q}{2\pi})\Big)$$

REY4 (14) *7/2022*

XYZ.2

$$\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \frac{e^{-p(x+z)-q(t+y)}\log^k\left(\frac{a(x+y)}{t+z}\right)}{\sqrt{t+z}\sqrt{x+y}}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t$$
$$= \frac{1}{pq(p-q)^2} i^k 2^{2k+1}\pi^{k+1}\Big((p+q)\zeta(-k,\frac{\pi-i\log a}{4\pi}) - (p+q)\zeta(-k,\frac{3\pi-i\log a}{4\pi})$$
$$+\sqrt{p}\sqrt{q}\Big[-\zeta(-k,\frac{-i\log a+i\log p-i\log q+\pi}{4\pi}) + \zeta(-k,\frac{-i\log a+i\log p-i\log q+3\pi}{4\pi})$$
$$-\zeta(-k,\frac{-i\log a-i\log p+i\log q+\pi}{4\pi}) + \zeta(-k,\frac{-i\log a-i\log p+i\log q+3\pi}{4\pi})\Big]\Big)$$

REY4 (15) 7/2022

XYZ.3

$$\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty (t+z)^{-m}(x+y)^{m-1}e^{-p(x+z)-q(t+y)}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t$$
$$= -\frac{\pi p^{-m-1}q^{-m-1}\csc(m\pi)(p^m-q^m)(qp^m-pq^m)}{(p-q)^2}$$

REY4 (16) 7/2022

XYZ.5

$$\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \frac{(t-x-y+z)}{\sqrt{t+z}(x+y)^{3/2}}e^{-p(x+z)-q(t+y)}\log^k\left(\frac{x+y}{t+z}\right)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t$$
$$= \frac{1}{p^{3/2}q^{3/2}(p-q)^2} i^k 2^{2k+1}\pi^{k+1}(p+q)\Big(-2\sqrt{p}\sqrt{q}\zeta(-k,\frac{1}{4}) + 2\sqrt{p}\sqrt{q}\zeta(-k,\frac{3}{4})$$
$$+p\zeta(-k,\frac{i\log p-i\log q+\pi}{4\pi}) - p\zeta(-k,\frac{i\log p-i\log q+3\pi}{4\pi})$$
$$+q\zeta(-k,\frac{-i\log p+i\log q+\pi}{4\pi}) - q\zeta(-k,\frac{-i\log p+i\log q+3\pi}{4\pi})\Big)$$

REY4 (18) 7/2022

1. On page xxxviii, add the following entry before "Weber function"

   $E_p(z)$ Exponential Integral 8.27

2. On page 220, add the following integral

2.641.13 $$\int \frac{x^2\cos(xb)}{a^2+x^2}\,e^{-cx^2}\,\mathrm{d}x$$
$$= \frac{\sqrt{\pi}}{2\sqrt{c}}e^{-b^2/(4c)} + \frac{a\pi}{4}e^{a^2c}\left\{-e^{-ab} - e^{ab} + \operatorname{erf}\left(\frac{2ac-b}{2\sqrt{c}}\right) + e^{ab}\operatorname{erf}\left(\frac{2ac+b}{2\sqrt{c}}\right)\right\}$$

DO

3. On page 247, add section 2.9 Other Elementary Functions

4. On page 247, add section 2.91 Minimum & Maximum

2.91.1 $\displaystyle \int\!\cdots\!\int_{[a,b]^n} f(\min x_i, \max x_i)\, d\mathbf{x} = n(n-1) \int_a^b dv \int_a^v f(u,v)(v-u)^{n-2}\, du$ MAR2007

2.91.2 $\displaystyle \int\!\cdots\!\int_{[a,b]^n} f(\mathbf{x}, \min x_i, \max x_i)\, d\mathbf{x}$

$$= \sum_{\substack{j,k=1\\ j\neq k}}^{n} \int_a^b dv \int_a^v du \int\!\cdots\!\int_{[u,v]^{n-2}} f\left(\mathbf{x}, u, v \mid x_j = u, x_k = v\right) \prod_{i\in[n]\setminus\{j,k\}} \mathrm{d}x_i$$

MAR2007

5. Add section 2.92 Floor Function

The floor of a number is the largest integer that is less than or equal to the number. For example $\lfloor 2.345 \rfloor = 2$ and $\lfloor 5 \rfloor = 5$.

2.92.1 $\displaystyle \underbrace{\int_0^1 \cdots \int_0^1}_{n} f\left(\lfloor x_1 + \cdots + x_n \rfloor\right) \mathrm{d}x_1 \cdots \mathrm{d}x_n = \sum_{k=0}^{n} \left\langle {n \atop k} \right\rangle \frac{f(k)}{n!}$ GR1994, #6.65, p 316, 557

where the $\left\langle {n \atop k} \right\rangle$ are Eulerian numbers

6. Add section 2.93 Fractional Part of Numbers

The fractional part of a number is $\{x\} = x - \lfloor x \rfloor$. For example $\{2.345\} = 0.345$ and $\{5\} = 0$.

2.93.1 $\displaystyle \int_a^{a+n} \{x\}\ \mathrm{d}x = \frac{n}{2} \qquad [a > 0, \quad n = 1, 2, 3, \dots]$ FUR2013, 2.42

2.93.2 $\displaystyle \int_0^1 \{kx\}\ \mathrm{d}x = \frac{1}{2} \qquad [k = 1, 2, 3, \dots]$ FUR2013, 2.28

2.93.3 $\displaystyle \int_0^1 \{nx\}^k\ \mathrm{d}x = \frac{1}{k+1} \qquad [k > -1, \quad n = 1, 2, 3, \dots]$ FUR2013, 2.44

2.93.4 $\displaystyle \int_0^1 (x - x^2)^k\, \{nx\}\ \mathrm{d}x = \frac{(k!)^2}{2(2k+1)!} \qquad [k = 0, 1, 2, \dots, \quad n = 1, 2, 3, \dots]$

FUR2013, 2.48

2.93.5 $\displaystyle \int_1^\infty \frac{\{x\}}{x^2}\, \mathrm{d}x = 1 - \boldsymbol{C}$ WOFP

2.93.6 $\displaystyle \int_1^\infty \frac{\{x\}}{x^{k+1}}\, \mathrm{d}x = \frac{1}{k-1} - \frac{\zeta(k)}{k} \qquad [k = 2, 3, 4, \dots]$ FUR2013, 2.9

2.93.7 $\displaystyle \int_1^\infty \frac{\{x\} - \frac{1}{2}}{x}\, \mathrm{d}x = -1 + \log(\sqrt{2\pi})$

2.93.8 $\displaystyle \int_0^1 \left(\{ax\} - \tfrac{1}{2}\right)\left(\{bx\} - \tfrac{1}{2}\right)\ \mathrm{d}x = \frac{1}{12ab} \qquad [\operatorname{Re} a > 0, \quad \operatorname{Re} b > 0]$ OLD

2.93.9 $\displaystyle \int_0^1 \left\{\frac{1}{x}\right\}\ \mathrm{d}x = 1 - \boldsymbol{C}$ WOFP

2.93.10 $\int_0^1 \left\{\frac{q}{x}\right\} \mathrm{d}x = \begin{cases} q(1-\boldsymbol{C}-\log q) & [0<q\le 1] \\ q\left(1+\frac{1}{2}+\cdots+\frac{1}{1+\lfloor q\rfloor}-\boldsymbol{C}-\log q+\frac{\lfloor q\rfloor(\{q\}-1)}{q(1+\lfloor q\rfloor)}\right) & [q>1] \end{cases}$ FUR2013, 2.5b

2.93.11 $\int_0^1 x^m \left\{\frac{1}{x}\right\} \mathrm{d}x = \frac{1}{m} - \frac{\zeta(m+1)}{m+1} \qquad [m>0]$ FUR2013, 2.20

2.93.12 $\int_0^1 \frac{x}{1-x} \left\{\frac{1}{x}\right\} \mathrm{d}x = \boldsymbol{C}$ FUR2013, 2.15

2.93.13 $\int_0^1 \left\{\frac{1}{x}\right\}^2 \mathrm{d}x = \log(2\pi) - 1 - \boldsymbol{C}$ QIN2011

2.93.14 $\int_0^1 \left\{\frac{k}{x}\right\}^2 \mathrm{d}x = k\left(\log(2\pi) - \boldsymbol{C} + 1 + \frac{1}{2} + \cdots + \frac{1}{k} + 2k\log k - 2k - 2\log k!\right)$ $[k=1,2,3,\dots]$ FUR2013, 2.6

2.93.15 $\int_0^1 \left\{\frac{1}{x}\right\}\left\{\frac{1}{1-x}\right\} \mathrm{d}x = 2\boldsymbol{C} - 1$ QIN2011

2.93.16 $\int_0^{1/2} \left\{\frac{1}{x}\right\}\left\{\frac{1}{1-x}\right\} \mathrm{d}x = \int_{1/2}^1 \left\{\frac{1}{x}\right\}\left\{\frac{1}{1-x}\right\} \mathrm{d}x = \boldsymbol{C} - \frac{1}{2}$ FUR2013, 2.10

2.93.17 $\int_0^1 \left\{\frac{1}{x}\right\}^2\left\{\frac{1}{1-x}\right\} \mathrm{d}x = \frac{5}{2} - \boldsymbol{C} - \log(2\pi)$ FUR2013, 2.12

2.93.18 $\int_0^1 \left\{\frac{1}{x}\right\}^2\left\{\frac{1}{1-x}\right\}^2 \mathrm{d}x = 4\log(2\pi) - 4\boldsymbol{C} - 5$ QIN2011

2.93.19 $\int_0^1 \left\{\frac{1}{x}\right\}^3\left\{\frac{1}{1-x}\right\}^3 \mathrm{d}x = 6\boldsymbol{C} + 2 - \zeta(2) - 3\log(2\pi) - \frac{18\zeta'(2)}{\pi^2}$ QIN2011

2.93.20 $\int_0^1 x^m \left\{\frac{1}{x}\right\}^m \mathrm{d}x = 1 - \frac{\zeta(2)+\zeta(3)+\cdots+\zeta(m+1)}{m+1}$ $[m=1,2,3,\dots]$ FUR2013, 2.21

2.94

2.94.1 $\int_0^1 \left\{\frac{1}{\sqrt[k]{x}}\right\} \mathrm{d}x = \frac{k}{k-1} - \zeta(k) \qquad [k=2,3,4,\dots]$ FUR2013, 2.7

2.94.2 $\int_0^1 \left\{\frac{k}{\sqrt[k]{x}}\right\} \mathrm{d}x = \frac{k}{k-1} - k^k\left(\zeta(k) - \frac{1}{1^k} - \frac{1}{2^k} \cdots - \frac{1}{k^k}\right) \qquad [k=2,3,4,\dots]$ FUR2013, 2.8

2.94.3 $\int_0^1 \left\{\frac{1}{k\sqrt[k]{x}}\right\} \mathrm{d}x = \frac{1}{k-1} - \frac{\zeta(k)}{k^k} \qquad [k=2,3,4,\dots]$ FUR2013, 2.9

2.95 Combination of fractional part and other functions

2.95.1 $\int_0^1 \left\{(-1)^{\lfloor\frac{1}{x}\rfloor}\frac{1}{x}\right\} \mathrm{d}x = 1 + \log\frac{2}{\pi}$ FUR2013, 2.13

2.95.2 $\int_0^1 x\left\{\frac{1}{x}\right\}\left\lfloor\frac{1}{x}\right\rfloor \mathrm{d}x = \frac{\pi^2}{12} - \frac{1}{2}$ FUR2013, 2.14a

2.95.3 $\int_0^1 \{\log x\}\, x^m \,\mathrm{d}x = \frac{e^{m+1}}{(m+1)(e^{m+1}-1)} - \frac{1}{(m+1)^2} \qquad [m > -1]$ FUR2013, 2.16

2.96 Multiple integrals

2.96.1 $\int_0^1 \int_0^1 \left\{k\frac{x}{y}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{k}{2}\left(1 + \frac{1}{2} + \cdots + \frac{1}{k} - \log k - \boldsymbol{C}\right) + \frac{1}{4}$
$[k = 1, 2, 3, \dots]$ FUR2013, 2.28

2.96.2 $\int_0^1 \int_0^1 \left\{\frac{mx}{ny}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{m}{2n}\left(\log\frac{n}{m} + \frac{3}{2} - \boldsymbol{C}\right)$
[m and n are integers with $m \le n$] FUR2013, 2.29

2.96.3 $\int_0^1 \int_0^1 \left\{\frac{x^k}{y}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{2k+1}{(k+1)^2} - \frac{\boldsymbol{C}}{k+1} \qquad [k \ge 0]$ FUR2013, 2.30

2.96.4 $\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\} \left(\frac{y}{x}\right)^k \mathrm{d}x\,\mathrm{d}y = 1 - \frac{\zeta(2) + \zeta(3) + \cdots + \zeta(k+1)}{2(k+1)}$
$[k = 1, 2, 3, \dots]$ FUR2013, 2.33

2.96.5 $\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\} \frac{y^k}{x^p}\, \mathrm{d}x\,\mathrm{d}y = \frac{1}{k-p+1} - \frac{\zeta(2) + \zeta(3) + \cdots + \zeta(k+1)}{(k+2-p)(k+1)}$
[k is an integer, p is real, $k - p > -1$] FUR2013, 2.34

2.96.6 $\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\} \left\{\frac{y}{x}\right\} \mathrm{d}x\,\mathrm{d}y = 1 - \frac{\pi^2}{12}$ FUR2013,2.36

2.96.7 $\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\}^2 \mathrm{d}x\,\mathrm{d}y = \frac{\log(2\pi)}{2} - \frac{1}{3} - \frac{\boldsymbol{C}}{2}$ FUR2013, 2.31

2.96.8 $\int_0^1 \int_0^1 x^m y^n \left\{\frac{x}{y}\right\} \left\{\frac{y}{x}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{1}{m+n+1}\left(\frac{1}{n+1} + \frac{1}{m+1} - \frac{\zeta(n+2)}{n+2} - \frac{\zeta(m+2)}{m+2}\right)$
$[m > -1, \quad n > -1]$ FUR2013, 2.37

2.96.9 $\int_0^1 \int_0^1 (xy)^n \left\{\frac{x}{y}\right\} \left\{\frac{y}{x}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{1}{(n+1)^2} - \frac{\zeta(n+1)}{(n+1)(n+2)}$
$[n > -1]$ FUR2013, 2.38

2.96.10 $\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\}^m \left\{\frac{y}{x}\right\}^m \mathrm{d}x\,\mathrm{d}y = 1 - \frac{\zeta(2) + \zeta(3) + \cdots + \zeta(m+1)}{m+1}$
$[m = 1, 2, 3, \dots]$ FUR2013, 2.40

2.96.11 $\int_0^1 \int_0^1 \left\{\frac{2x}{y}\right\} \left\{\frac{2y}{x}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{49}{6} - \frac{2\pi^2}{3} - 2\log 2$ FUR2013, 2.39

2.96.12 $\int_0^1 \int_0^1 \left\{\frac{x-y}{x+y}\right\} \mathrm{d}x\,\mathrm{d}y = \int_0^1 \int_0^1 \left\{\frac{x+y}{x-y}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{1}{2}$ FUR2013, 2.51

2.96.13 $\int_0^1 \int_0^1 \left\{\frac{k}{x-y}\right\} \left\{\frac{1}{x}\right\} \left\{\frac{1}{y}\right\} \mathrm{d}x\,\mathrm{d}y = \frac{1}{2}(1 - \boldsymbol{C})^2 \qquad [k > 0]$ FUR2013, 2.52

2.96.14 $\int_0^1 \int_0^1 x \left\{\frac{1}{1-xy}\right\} \mathrm{d}x\,\mathrm{d}y = 1 - \frac{\zeta(2)}{2} = 1 - \frac{\pi^2}{12}$ FUR2013, 2.23

2.96.15 $\int_0^1 \int_0^1 \left\{\frac{1}{x+y}\right\}^m \mathrm{d}x\,\mathrm{d}y = \begin{cases} 2\log 2 - \frac{\pi^2}{12} & m = 1 \\ \frac{5}{2} - \log 2 - \boldsymbol{C} - \frac{\pi^2}{12} & m = 2 \end{cases}$ FUR2013, 2.24

2.96.16 $\displaystyle\iint_{0\le x,y\le 1} \left\{\frac{1}{x+y}\right\}^m \mathrm{d}x\,\mathrm{d}y = \begin{cases} 2\log 2 - \frac{\pi^2}{12} & m=1 \\ \frac{3}{2} - \frac{\pi^2}{12} - \log 2 - \boldsymbol{C} & m=2 \end{cases}$ QIN2011, 3.1

2.96.17 $\displaystyle\int\int\int_{0\le x,y,z\le 1} \left\{\frac{1}{x+y+z}\right\}^m \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \begin{cases} \frac{9}{2}\log 3 - \frac{13}{24} - \frac{19}{4}\log 2 - \frac{\zeta(3)}{3} & m=1 \\ \frac{53}{24} + 4\log 2 - 3\log 3 - \frac{\zeta(3)}{3} - \frac{\pi^2}{12} & m=2 \end{cases}$ QIN2011, 3.2

2.96.18 $\displaystyle\int_0^{a_1} \cdots \int_0^{a_n} \{k(x_1+x_2+\cdots+x_n)\}\,\mathrm{d}x_n \cdots \mathrm{d}x_1 = \frac{1}{2}a_1a_2\cdots a_n$ FUR2013, 2.42b

7. On page 572, add the following integral

4.318.3 $\displaystyle\int_0^1 \frac{\log[(1+x^a)(1+x^{1/a})]}{1+x}\,\mathrm{d}x = (\log 2)^2 \qquad [a>0]$

8. On page 574, add the following integrals

4.325.13 $\displaystyle\int_0^1 \frac{\log(\log x)}{1+x^2}\,\mathrm{d}x = \frac{\pi}{4}\left(i\pi + \log\left(\frac{4\pi^3}{\Gamma^4\left(\frac{1}{4}\right)}\right)\right)$

4.325.14 $\displaystyle\int_0^\infty \frac{\log(\log x)}{1+x^2}\,\mathrm{d}x = \frac{\pi}{4}\left(i\pi + \log\left(\frac{4\pi^2\Gamma^4\left(\frac{3}{4}\right)}{\Gamma^4\left(\frac{1}{4}\right)}\right)\right)$ REY1 (13)

(Thanks to Robert Reynolds for suggesting the inclusion of these evaluations.)

9. On page 583, add the following integrals

4.374.3 $\displaystyle\int_0^\infty \ln(1+x^2)\ln\left(\tanh\left(\frac{\pi x}{4}\right)\right)\,\mathrm{d}x = \pi - 4G$ REY3 (18)

Here, $G \approx 0.915$ is Catalan's constant.

4.374.4 $\displaystyle\int_0^\infty \frac{\ln\left(\tanh\frac{ax}{2}\right)}{b^2+x^2}\,\mathrm{d}x = \frac{\pi}{2b}\ln\left(\frac{ab}{2\pi}\frac{\Gamma\left(\frac{\pi+ab}{2\pi}\right)}{\Gamma\left(\frac{2\pi+ab}{2\pi}\right)}\right) \qquad [\operatorname{Re} a>0, \operatorname{Re}>0]$ REY3 (19)

4.374.5 $\displaystyle\int_0^\infty \frac{1-3x^2}{(1+x^2)^3}\ln\left(\tanh\frac{\pi x}{4}\right)\,\mathrm{d}x = \frac{\pi}{4}(1-2G)$ REY3 (20)

Here, $G \approx 0.915$ is Catalan's constant.

(Thanks to Robert Reynolds for suggesting the inclusion of these evaluations.)

10. On page 611, add the following integrals

4.592.1 $\displaystyle\int_0^\infty \frac{\arctan(x)}{x\log^2(-x)}\,\mathrm{d}x = -i\frac{4-\pi}{2}$ REY2 (11)

4.592.2 $\displaystyle\int_0^\infty \frac{\arctan(x)}{x\log^3(-x)}\,\mathrm{d}x = 2\frac{C-1}{\pi}$ REY2 Table

Here, C is Catalan's constant.

4.592.3 $\displaystyle\int_0^\infty \frac{\arctan(x)}{x\log^2(ix)}\,\mathrm{d}x = -i\log 2$ REY2 (15)

4.592.4 $\displaystyle\int_0^\infty \frac{\arctan(x)}{x\log^3(ix)}\,\mathrm{d}x = -\frac{\pi}{24}$ REY2 Table

(Thanks to Robert Reynolds for suggesting the inclusion of these evaluations.)

11. On page 617, add the following integral

4.626.16 $$\int_0^1\int_0^1 \left[-\log(1-st)\right]^n \,\mathrm{d}t\,\mathrm{d}s = (n+1)! - n!\sum_{j=2}^{n+1}\zeta(j)$$ BUS

for $n = 0, 1, 2, \ldots$ and $\zeta()$ is the Riemann zeta function.

12. On page 617, just before 6.631, add the following text:

See also: 2.91.1, 2.91.2, 2.92.1, 2.96.18

13. On page 622, add new section 4.65 Multiple integrals of exponentials of linear functions

Notation: $k = |\mathbf{k}|$, $r = |\mathbf{r}|$, $\widehat{\mathbf{k}} = \dfrac{\mathbf{k}}{|\mathbf{k}|}$, $\widehat{\mathbf{r}} = \dfrac{\mathbf{r}}{|\mathbf{r}|}$, and $\displaystyle\int \mathrm{d}^n k = \int_{\mathbb{R}^n} \mathrm{d}\mathbf{k} = \underbrace{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{n} \mathrm{d}k_1\cdots\mathrm{d}k_n$.

4.65.1 $$\int \frac{\mathrm{d}^n k}{(2\pi)^n} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} = \delta^n(\mathbf{x}-\mathbf{y})$$ WIKIQ

4.65.2 $$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2} = \frac{1}{4\pi r}$$ WIKIQ

4.65.3 $$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2+m^2} = \frac{e^{-mr}}{4\pi r}$$ WIKIQ

4.65.4 $$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left(\widehat{\mathbf{k}}\cdot\widehat{\mathbf{r}}\right)^2 \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2+m^2} = \frac{e^{-mr}}{4\pi r}\left[1 + \frac{2}{mr} - \frac{2}{(mr)^2}\left(e^{mr}-1\right)\right]$$ WIKIQ

4.65.5 $$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[\widehat{\mathbf{k}\mathbf{k}}\right] \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2+m^2} = \frac{1}{2}\frac{e^{-mr}}{4\pi r}\left((\mathbf{1} - \widehat{\mathbf{r}\mathbf{r}}) + \left[1 + \frac{2}{mr} - \frac{2}{(mr)^2}\left(e^{mr}-1\right)\right](\mathbf{1} + \widehat{\mathbf{r}\mathbf{r}})\right)$$ WIKIQ

4.65.6 $$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[\mathbf{1} - \widehat{\mathbf{k}\mathbf{k}}\right] \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2+m^2} = \frac{1}{2}\frac{e^{-mr}}{4\pi r}\left[-\frac{2}{mr} + \frac{2}{(mr)^2}\left(e^{mr}-1\right)\right](\mathbf{1} + \widehat{\mathbf{r}\mathbf{r}})$$ WIKIQ

4.65.7 $$\int_{\mathbb{R}^n} e^{i\mathbf{x}\cdot\mathbf{r}} \frac{\sin\left[t\sqrt{r^2+m^2}\right]}{\sqrt{r^2+m^2}}\,\mathrm{d}\mathbf{r}$$ GLAS
$$= \pi^{(n+1)/2}\left(\frac{m}{2}\right)^{(n-1)/2}\left(t^2-k^2\right)^{(1-n)/4} J_{(1-n)/2}\left(m\sqrt{t^2-k^2}\right) H(t-k)$$

where $r = |\mathbf{r}|$, $k = |\mathbf{x}|$, H is the unit step function, and $0 < n < 3$.

14. On page 622, add new section 4.66 Multiple integrals of exponentials of powers

Notation: $\displaystyle\int \mathrm{d}^n x = \int_{\mathbb{R}^n} \mathrm{d}\mathbf{x} = \underbrace{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{n} \mathrm{d}x_1\cdots\mathrm{d}x_n$.

4.66.1 $\displaystyle\int_{\mathbb{R}^n} \exp\left(-\frac{1}{2}\mathbf{x}^{\mathrm{T}}A\mathbf{x} + \mathbf{b}^{\mathrm{T}}\mathbf{x}\right)\,\mathrm{d}\mathbf{x} = \sqrt{\frac{(2\pi)^n}{\det A}}\exp\left(\frac{1}{2}\mathbf{b}^{\mathrm{T}}A^{-1}\mathbf{b}\right)$ WIKIQ

4.66.2 $\displaystyle\int_{\mathbb{R}^n} \exp\left(-\frac{1}{2}\mathbf{x}^{\mathrm{T}}A\mathbf{x} + i\mathbf{b}^{\mathrm{T}}\mathbf{x}\right)\,\mathrm{d}\mathbf{x} = \sqrt{\frac{(2\pi)^n}{\det A}}\exp\left(-\frac{1}{2}\mathbf{b}^{\mathrm{T}}A^{-1}\mathbf{b}\right)$ WIKIQ

4.66.3 $\displaystyle\int_{\mathbb{R}^n} \exp\left(-\frac{i}{2}\mathbf{x}^{\mathrm{T}}A\mathbf{x} + i\mathbf{b}^{\mathrm{T}}\mathbf{x}\right)\,\mathrm{d}\mathbf{x} = \sqrt{\frac{(2\pi i)^n}{\det A}}\exp\left(-\frac{i}{2}\mathbf{b}^{\mathrm{T}}A^{-1}\mathbf{b}\right)$ WIKIQ

15. On page 622, add new section 4.70 Multiple integrals and the omega calculus

In the Omega calculus, the Omega operator is defined by $\underset{=}{\overset{\boldsymbol{\lambda}}{\Omega}} \displaystyle\sum_{a_1=-\infty}^{\infty} \cdots \sum_{a_n=-\infty}^{\infty} A_{\boldsymbol{\alpha}}\boldsymbol{\lambda}^{\boldsymbol{\alpha}} = A_0$ when $A_{\boldsymbol{\alpha}} \in \mathbb{C}^{n\times n}$ for $\boldsymbol{\alpha} \in \mathbb{Z}^n$ and $\boldsymbol{\lambda}^{\boldsymbol{\alpha}} = \lambda_1^{\alpha_1}\cdots\lambda_n^{\alpha_n}$,

4.70.1 $$\int_0^1\int_0^{s_1}\cdots\int_0^{s_{k-2}} e^{(t-s_1)A_{1,1}}A_{1,2}\,e^{(s_1-s_2)A_{2,2}}A_{2,3}\,e^{(s_2-s_3)A_{3,3}}A_{3,4}\cdots e^{(s_{k-2}-s_{k-1})A_{k-1,k-1}}A_{k-1,k}\,e^{s_{k-1}A_{k,k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{k-1}$$ $$= \underset{=}{\overset{\mu}{\Omega}}\, e^{\mu t}B_{1,1}\frac{A_{1,2}}{\mu}B_{2,2}\frac{A_{2,3}}{\mu}B_{3,3}\cdots B_{k-1,k-1}\frac{A_{k-1,k}}{\mu}B_{k,k}$$ NETO

where A_{ij} are matrices, $B_{i,i} = \left(I - \dfrac{A_{i,i}}{\mu}\right)^{-1}$, and $\underset{=}{\overset{\boldsymbol{\lambda}}{\Omega}}$ is the Omega operator.

4.70.2 $\displaystyle\int_0^t e^{sA}B\,\mathrm{d}s = \underset{=}{\overset{\mu}{\Omega}}\, e^{\mu t}\left(I - \frac{A}{\mu}\right)^{-1}\frac{B}{\mu}$ where A and B are matrices NETO

16. On page 632, add new section 5.24 Generalized Exponential Integral

17. On page 632, add new section 5.24.1 General Index

Add the following integrals

5.24.1.1 $\displaystyle\int_z^\infty E_{p-1}(t)\,\mathrm{d}t = E_p(z) \qquad [|\arg z| < \pi]$ DLMF 8.19.23

5.24.1.2 $\displaystyle\int_0^\infty e^{-ax}\,E_n(x)\,\mathrm{d}x = \frac{(-1)^{n-1}}{a^n}\left(\ln(1+a) + \sum_{k=1}^{n-1}\frac{(-1)^k a^k}{k}\right)$ $[n = 1, 2, \ldots, \quad \operatorname{Re} a > -1]$ DLMF 8.19.24

5.24.1.3 $\displaystyle\int_0^\infty e^{-ax}x^{b-1}\,E_p(x)\,\mathrm{d}x = \frac{\Gamma(b)(1+a)^{-b}}{p+b-1}F\left(1, b; p+b; \frac{a}{1+a}\right)$ $[\operatorname{Re} a > -1, \quad \operatorname{Re}(p+b) > 1]$ DLMF 8.19.25

5.24.1.4 $\displaystyle\int_0^\infty E_p(x)\,E_q(x)\,\mathrm{d}x = \frac{L(p)+L(q)}{p+q-1} \qquad [p > 0, \quad q > 0, \quad p+q > 1]$

where $\displaystyle L(p) = \int_0^\infty e^{-t}\,E_p(t)\,\mathrm{d}t = \frac{1}{2p}F(1, 1; 1+p; \tfrac{1}{2})$ DLMF 8.19.26

5.24.1.5 $\displaystyle\int_0^z x^\lambda\,E_\nu(x^\mu)\,\mathrm{d}x = \frac{\gamma\left(\frac{1+\lambda}{\mu}, z^\mu\right) + z^{1+\lambda}\,E_\nu(z^\mu)}{1+\lambda+\mu(\nu-1)}$ $[\mu > 0, \quad z \geq 0, \quad \lambda > \max(-1, -1-\mu(\nu-1))]$ CIO2020

18. On page 632, add new section 5.24.2 Indefinite Exponential Integrals of Index 1

Add the following indefinite integrals

5.24.2.1 $\int E_1(ax)\,dx = x\,E_1(ax) - \frac{1}{a}e^{-ax} \qquad [a>0]$ GEL (4.1-1)

5.24.2.2 $\int x\,E_1(ax)\,dx = \frac{1}{2}x^2\;E_1(ax) - \frac{1}{2a^2}(1+ax)e^{-ax} \qquad [a>0]$ GEL (4.1-4)

5.24.2.3 $\int x^p\,E_1(ax)\,dx = \frac{x^{p+1}}{p+1}\;E_1(ax) + \frac{1}{(p+1)a^{p+1}}\gamma(p+1,ax) \qquad [a>0, \quad p>-1]$ GEL (4.1-14)

5.24.2.4 $\int e^{-ax}\;E_1(bx)\,dx = \frac{1}{a}\left(E_1[(a+b)x] - e^{-ax}\;E_1(bx)\right)$ $[a>0, \quad b>0]$ GEL (4.2-1)

5.24.2.5 $\int e^{ax}\;E_1(bx)\,dx = -\frac{1}{a}\left(E_1[(b-a)x] - e^{ax}\;E_1(bx)\right) \qquad [b>a>0]$ GEL (4.2-2)

5.24.2.6 $\int xe^{-ax}\;E_1(bx)\,dx = \frac{1}{a^2}\left(E_1[(a+b)x] - (1+ax)e^{-ax}\;E_1(bx) + \left(\frac{a}{a+b}\right)e^{-(a+b)x}\right)$ $[a,b,c>0]$ GEL (4.2-10)

5.24.2.7 $\int xe^{cx}\;E_1(ax+b)\,dx = \frac{1}{c}\left(x-\frac{1}{c}\right)e^{cx}\;E_1(ax+b) - \frac{e^{(a-c)x+b}}{c(a-c)} + \frac{(a+bc)e^{-bc/a}}{ac^2}\;E_1\left(\frac{(a-c)(ax+b)}{a}\right)$ $[a>c>0, \quad b>0]$ GEL (4.2-13)

5.24.2.8 $\int \frac{e^{-x}}{x}\;E_1(ax)\,dx = -\frac{1}{2}\left[E_1(ax)\right]^2 \qquad [a>0]$ GEL (4.2-30)

5.24.2.9 $\int \ln x\;E_1(ax)\,dx = \frac{1}{a}\left[(1-\ln x)e^{-ax} - (1+ax-ax\ln x)\;E_1(ax)\right]$ $[a>0]$ GEL (4.4-1)

5.24.2.10 $\int E_1(ax)\;E_1(bx)\,dx = x\;E_1(ax)\;E_1(bx) + \left(\frac{1}{a}+\frac{1}{b}\right)E_1\left([a+b]x\right) - \frac{1}{a}e^{-ax}\,E_1(bx) - \frac{1}{b}e^{-bx}\;E_1(ax)$ $[a,b>0]$ GEL (4.6-1)

19. On page 632, add new section 5.24.3 Definite Exponential Integrals of Index 1

Add the following definite integrals

5.24.3.1 $\int_0^\infty E_1(ax)\,dx = \frac{1}{a} \qquad [a>0]$ GEL (4.1-3)

5.24.2.2 $\int_0^\infty x^p\;E_1(ax)\,dx = \frac{\Gamma(p+1)}{(p+1)a^{p+1}} \qquad [a>0, \quad p>-1]$ GEL (4.1-15)

5.24.2.3 $\int_0^\infty e^{-ax}\;E_1(bx)\,dx = \frac{1}{a}\ln\left(1+\frac{a}{b}\right) \qquad [a>0, \quad b>0]$ GEL (4.2-3)

5.24.2.4 $\int_0^\infty e^{ax}\;E_1(bx)\,dx = -\frac{1}{a}\ln\left(1-\frac{a}{b}\right) \qquad [b>a>0]$ GEL (4.2-4)

5.24.2.5 $\int_0^\infty x^p e^{ax}\;E_1(bx)\,dx = \frac{\Gamma(p+1)}{b^{p+1}(p+1)}\,{}_2F_1\left(p+1,p+1;p+2;\frac{a}{b}\right)$ $[b>a>0, \quad p>-1]$ GEL (4.2-21)

5.24.2.6 $$\int_{-\infty}^{\infty} e^{ax} e^{-ibx}\, E_1(ax)\, \mathrm{d}x = \frac{b\pi}{b+ia} \qquad [a, b > 0]$$ GEL (4.2-34)

5.24.2.7 $$\int_0^{\infty} x^n \ln x\, E_1(ax)\, \mathrm{d}x = -\frac{n!}{(n+1)a^{n+1}} \left[\gamma + \ln a + \frac{1}{n+1} - \sum_{m=1}^{n} \frac{1}{m}\right]$$ $[n = 1, 2, \ldots, \quad a > 0]$ GEL (4.4-7)

5.24.2.8 $$\int_0^{\infty} \sin(bx) e^{cx}\, E_1(ax)\, \mathrm{d}x = \frac{1}{b^2+c^2}\left[\frac{b}{2}\ln\left(\frac{(a-c)^2+b^2}{a^2}\right) + c\tan^{-1}\left(\frac{b}{a-c}\right)\right]$$ $[a \geq c > 0, \quad b > 0]$ GEL (4.3-10)

5.24.2.9 $$\int_0^{\infty} \cos(bx) e^{cx}\, E_1(ax)\, \mathrm{d}x = \frac{1}{b^2+c^2}\left[-\frac{c}{2}\ln\left(\frac{(a-c)^2+b^2}{a^2}\right) + b\tan^{-1}\left(\frac{b}{c-a}\right)\right]$$ $[a \geq c > 0, \quad b > 0]$ GEL (4.3-13)

5.24.2.10 $$\int_0^{\infty} x\, E_1(ax)\, E_1(bx)\, \mathrm{d}x = \frac{1}{2}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)\ln(a+b) - \frac{1}{2a^2}\ln b - \frac{1}{2b^2}\ln a - \frac{1}{2ab}$$ $[a, b > 0]$ GEL (4.6-8)

5.24.2.11 $$\int_0^{\infty} E_1(x)\, J_0(ax)\, \mathrm{d}x = \frac{\operatorname{arcsinh} a}{a} \qquad [a > 0]$$ GEL (4.7-1)

20. On page 634, add new section 5.57 and include the following integrals

5.57.1 $$\int \frac{\mathrm{d}x}{x\, J_p^2(x)} = \frac{\pi}{2}\frac{Y_p(x)}{J_p(x)}$$ WA (pg 133)

5.57.2 $$\int \frac{\mathrm{d}x}{x\, J_p(x)\, Y_p(x)} = \frac{\pi}{2}\log\frac{Y_p(x)}{J_p(x)}$$ WA (pg 133)

5.57.3 $$\int \frac{\mathrm{d}x}{x\, Y_p^2(x)} = -\frac{\pi}{2}\frac{J_p(x)}{Y_p(x)}$$ WA (pg 133)

(Thanks to Brady Metherall for suggesting the inclusion of these evaluations.)

21. On page 635, create new section 5.8 Lambert W-function 7/2022

5.8.1 $$\int \mathrm{W}(x)\, \mathrm{d}x = x\,\mathrm{W}(x) - x + e^{\mathrm{W}(x)}$$ COR1 (3.14)

5.8.2 $$\int x\,\mathrm{W}(x)\, \mathrm{d}x = \tfrac{1}{2}\left(\mathrm{W}(x) - \tfrac{1}{2}\right)\left(\mathrm{W}^2(x) + \tfrac{1}{2}\right)e^{2\,\mathrm{W}(x)}$$ COR1 (3.15)

5.8.3 $$\int \frac{\mathrm{W}(x)}{x}\, \mathrm{d}x = \int e^{-\mathrm{W}(x)}\, \mathrm{d}x = \tfrac{1}{2}\mathrm{W}^2(x) + \mathrm{W}(x)$$

5.8.4 $$\int \frac{\mathrm{W}(x)}{x^2}\, \mathrm{d}x = \mathrm{Ei}(-\mathrm{W}(x)) - e^{-\mathrm{W}(x)}$$

5.8.5 $$\int x\sin\left(\mathrm{W}(x)\right)\, \mathrm{d}x = \tfrac{1}{2}\left(x + \frac{x}{\mathrm{W}(x)}\right)\sin\left(\mathrm{W}(x)\right) - \frac{x}{2}\cos\left(\mathrm{W}(x)\right)$$ COR2

5.8.6 $$\int \mathrm{W}(ae^{bx})\, \mathrm{d}x = \frac{\mathrm{W}(ae^{bx})^2}{2b} + \frac{\mathrm{W}(ae^{bx})}{b}$$

(Thanks to Brady Metherall for suggesting the inclusion of many of these evaluations.)

22. On page 640, add the following integrals

6.142.3 $$\int_0^1 \frac{k\,\mathbf{K}(k)}{(z+k^2)^{n+3/2}}\,dk = \frac{(-2)^n}{(2n+1)!!}\frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(\frac{\operatorname{arccot}(\sqrt{z})}{\sqrt{z(z+1)}}\right)$$

$z>0$ and $n=0,1,2,\ldots$ CIO2019

6.142.4 $$\int_0^1 \frac{k\,\mathbf{K}(k)}{(1+k^2)^{3/2}}\,dk = \frac{\pi}{4\sqrt{2}}$$ CIO2019

6.142.5 $$\int_0^1 \frac{k\,\mathbf{K}(k)}{(1+k^2)^{5/2}}\,dk = \frac{4+3\pi}{24\sqrt{2}}$$ CIO2019

6.142.6 $$\int_0^1 \frac{k\,\mathbf{K}(k)}{(1+k^2)^{7/2}}\,dk = \frac{40+19\pi}{240\sqrt{2}}$$ CIO2019

6.142.7 $$\int_0^1 \frac{k\,\mathbf{K}(k)}{(1+k^2)^{9/2}}\,dk = \frac{484+189\pi}{3360\sqrt{2}}$$ CIO2019

(Thanks to Luca Ciotti for suggesting the inclusion of these integrals.)

23. On page 680, add the following two additional evaluations to 6.541:

$$= \frac{\delta(b-a)}{a} - c^2\, I_\nu(bc)\, K_\nu(ac) \qquad [n=-1, \nu=0,1,\ldots, \operatorname{Re} c>0, 0<b<a]$$

$$= \frac{\delta(b-a)}{a} - c^2\, I_\nu(ac)\, K_\nu(bc) \qquad [n=-1, \nu=0,1,\ldots, \operatorname{Re} c>0, 0<a<b]$$

(Thanks to Peter J. Hobson for suggesting the inclusion of these evaluations.)

24. On page 715, add the following integrals:

6.633.6 $$\int_0^\infty x e^{-ax^2}\, J_1(x)\, Y_1(x)\,\mathrm{d}x = -\frac{1}{\pi} + \frac{e^{-1/2a}}{2a\pi}\, K_1\left(\frac{1}{2a}\right)$$ MCP (19)

6.633.7 $$\int_0^\infty x e^{-ax^2}\, J_2(x)\, Y_2(x)\,\mathrm{d}x = -\frac{2(1-2a)}{\pi} - \frac{e^{-1/2a}}{2a\pi}\, K_2\left(\frac{1}{2a}\right)$$ MCP (20)

25. On page 717, add the following integral

6.645.4 $$\int_{-1}^1 \left(1-x^2\right)^{\frac{1}{2}\nu} e^{-\alpha x} I_\nu\left(\beta\sqrt{1-x^2}\right)\,\mathrm{d}x = \sqrt{2\pi}\beta^\nu\left(\alpha^2+\beta^2\right)^{-\frac{1}{2}\nu-\frac{1}{4}} I_{\nu+\frac{1}{2}}\left(\sqrt{\alpha^2+\beta^2}\right)$$

$[\nu \geq 0]$

(Thanks to Christoph Gierull for suggesting the inclusion of this integral.)

26. On page 726, create a new section 6.6711

27. On page 726, add the following integral

6.6711.1 $$\int_0^1 J_0(ax)\arccos(x)\,\mathrm{d}x = \frac{\operatorname{Si}(a)}{a} \qquad [\operatorname{Re}(a)>0]$$ MA

(Thanks to Luca Ciotti for suggesting the inclusion of this integral.)

28. On page 738, add the following extra cases to the existing integrals

6.699.1 $\text{integral} = \dfrac{2^{\nu-1}\Gamma\left(-\frac{1}{2}-\lambda\right)\Gamma\left(\frac{3}{2}+\frac{1}{2}\lambda+\frac{1}{2}\nu\right)}{a^{\lambda+1}\Gamma\left(\nu-\lambda\right)\Gamma\left(\frac{1}{2}-\frac{1}{2}\lambda-\frac{1}{2}\nu\right)}$

$[b = a, \quad a > 0, \quad -1 < \operatorname{Re}\nu < \operatorname{Re}(1+\lambda) < \frac{1}{2}]$ ET 1 6.8(10)

6.699.2 $\text{integral} = \dfrac{2^{\nu-1}\Gamma\left(-\frac{1}{2}-\lambda\right)\Gamma\left(1+\frac{1}{2}\lambda+\frac{1}{2}\nu\right)}{a^{\lambda+1}\Gamma\left(\frac{1}{2}\lambda-\frac{1}{2}\nu\right)\Gamma\left(\nu-\lambda\right)}$

$[b = a, \quad a > 0, \quad -\operatorname{Re}\nu < \operatorname{Re}(1+\lambda) < \frac{1}{2}]$ ET 1 6.8(11)

(Thanks to Shenhui Liu for suggesting the inclusion of these evaluations.)

29. Add section 7.9 Lambert W-function

7.9.1 $\displaystyle\int_0^\infty \mathrm{W}(x)x^{-3/2}\,\mathrm{d}x = \sqrt{8\pi}$

7.9.2 $\displaystyle\int_0^e \mathrm{W}(x)\,\mathrm{d}x = e-1$

7.9.3 $\displaystyle\int_0^e \frac{x}{\mathrm{W}(x)}\,\mathrm{d}x = \frac{3e^2}{4}$

30. On page 900, add section 8.27: Generalized Exponential Integral

8.271 Definition

8.271.1 $E_p(z) = z^{p-1}\Gamma(1-p,z) = z^{p-1}\displaystyle\int_z^\infty \frac{e^{-t}}{t^p}\,\mathrm{d}t$ DLMF 8.19.2

8.271.2 $E_p(z) = \displaystyle\int_a^\infty \frac{e^{-zt}}{t^p}\,\mathrm{d}t \qquad [|\arg z| < \tfrac{1}{2}\pi]$ DLMF 8.19.3

8.271.3 $E_p(z) = \dfrac{z^{p-1}e^{-z}}{\Gamma(p)}\displaystyle\int_0^\infty \frac{t^{p-1}e^{-zt}}{1+t}\,\mathrm{d}t \qquad [|\arg z| < \tfrac{1}{2}\pi, \quad \operatorname{Re}p > 0]$ DLMF 8.19.3

8.271.4 $E_0(z) = z^{-1}e^{-z} \qquad [z \neq 0]$ DLMF 8.19.5

8.271.5 $E_p(0) = \dfrac{1}{p-1} \qquad [\operatorname{Re}p > 1]$ DLMF 8.19.6

8.271.6 $E_1(-x \pm i0) = -\mathrm{Ei}(x) \mp i\pi$ DLMF 6.5.1

8.271.7 $E_p(x) = \begin{cases} \dfrac{e^{-x} - x\,E_{p-1}(x)}{p-1} & p \neq 1 \\[2ex] \dfrac{e^{-x} - p\,E_{p+1}(x)}{z} & \end{cases}$

31. On page 945, add section 8.5181: The series $\sum J_{k+\nu}(x)J_{k+\mu}(x)$

8.5181.1 $\displaystyle\sum_{k=0}^\infty J_{k+\nu}(x)J_{k+\mu}(x) = K(\mu,\nu) \qquad [\mu \text{ and } \nu \text{ are real}]$

$$K(\mu,\nu) = \frac{(x/2)^{\mu+\nu}}{\Gamma(\mu+1)\Gamma(\nu+1)}\,{}_2F_3\left[\frac{\mu+\nu}{2}, \frac{\mu+\nu+1}{2}; \mu+1, \nu+1, \mu+\nu; -x^2\right]$$

8.5181.2 $\sum_{k=L}^{M} J_{k+\nu}(x)J_{k+\mu}(x) = K(\mu+L,\nu+L) - K(\mu+M+1,\nu+M+1)$

8.5181.3 $\sum_{k=0}^{\infty} J_k(x)J_{k+\mu}(x) = \frac{x}{2\mu}\left[J_0(x)J_{\mu-1}(x) + J_1(x)J_\mu(x)\right]$ [μ is real]

8.5181.4 $\sum_{k=0}^{\infty} J_k(x)J_{k+1}(x) = \frac{x}{2}\left[J_0^2(x) + J_1^2(x)\right]$

8.5181.5 $\sum_{k=0}^{\infty} J_k(x)J_{k+2}(x) = \frac{x}{4}\left[J_0(x)J_1(x) + J_1(x)J_2(x)\right] = \frac{1}{2}J_1^2(x)$

8.5181.6 $\sum_{k=0}^{\infty} J_k(x)J_{k+3}(x) = \frac{x}{6}\left[J_0(x)J_2(x) + J_1(x)J_3(x)\right]$

(Thanks to David A. Kessler for suggesting the inclusion of these sums.)

32. On page 945, add section 8.5182 The series $\sum a_k J_k^2(x)$

8.5182.1 $\sum_{k=0}^{\infty} k\, J_k^2(x) = \frac{x^2}{2}\left[J_0^2(x) + J_1^2(x)\right] - \frac{x}{2}J_0(x)J_1(x)$

8.5181.2 $\sum_{k=0}^{\infty} k^2\, J_k^2(x) = \frac{x^2}{4}$

(Thanks to David A. Kessler for suggesting the inclusion of these sums.)

33. On pages 1105–1108, add the following references:

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- WIKIW Wikipedia contributors, "Lambert W function," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=Lambert_W_function&oldid=1019399859, (accessed April 20, 2021).
- WOFP Wolfram, *Fractional Part*, https://mathworld.wolfram.com/FractionalPart.

`html`, (accessed April 20, 2021).

ERRATA FOR THE 8th EDITION

1. On pages xix–xxiii, **Acknowledgements**, add the following names:

- Mohammad S. Alhassoun
- Dominik Beck
- Dr. Elliot Blackstone
- Dr. Sam Blake
- Dr. Guillem Blanco
- Dr. Farid Bouttout
- Dr. Adriana Brancaccio
- Peter Brown
- Dr. Patrick Bruno
- Dr. Michele Cappellari
- Dr. Luca Ciotti
- Mark Coffey
- Bruno Daniel
- Dr. Christophe De Beule
- Dr. Gerald Edgar
- Dr. Joseph Gangestad
- Dr. Howard Haber
- Mariam Mousa Harb
- Dr. Aaron Hendrickson
- Dr. Peter J. Hobson
- Richard Hunt
- Dr. Ramakrishna Janaswamy
- Dr. David A. Kessler
- Martin Kreh
- Leland Langston
- Dr. Mo Li
- Dr. Wenzhi Luo
- Dr. Matt Majic
- Dr. Brady Metherall
- Dr. J. P. Balthasar Müller
- Dr. Travis Porco
- Steven Reyes
- Dr. Robert Reynolds
- Lasse Schmieding
- Dr. Allan Stauffer
- Dr. Allen Stenger
- Claudio Severi
- Dr. Michael James Ungs
- Dr. Martin Venker
- Dr. Michal Wierzbicki
- Dr. Hongjun Xiang
- Dr. Shotaro Yamazoe
- Dr. Junggi Yoon
- The Bogazici Physics seniors of 2018

   (a) The name "Dr. M. A. F. Sanjun" is incorrect; it should be "Dr. Miguel A. F. Sanjuan."
   (b) The name "Dr. D. Rudermann" is incorrect; it should have been "Dr. Dan Ruderman."
   (c) The name "Richard Marthar" should be removed. The correct spelling ("Richard J. Mathar") is already present.

2. Page xxxvii, Index of Special Functions: After the ψ entry, add the following entry

   $\Psi(\alpha, \gamma; z)$ Confluent hypergeometric function 9.210

   (Thanks to Lasse Schmieding for correcting this error.)

3. Page 9, Formula 0.232.3 the entire right hand side should be multiplied by b^a. That is the evaluation begins $\frac{b^a}{(b-1)^{a+1}} \sum_{i=1}^{a}$

   (Thanks to J. P. Balthasar Müller for correcting this error.)

4. Page 24, Formula 0.435: replace $\frac{\mathrm{d}^n(y^3)}{\mathrm{d}x^n} x^n$ with $\frac{\mathrm{d}^n(y^3)}{\mathrm{d}x^n}$

   (Thanks to Steven Reyes for correcting this error.)

5. Page 32, Formula 1.323.6: replace "cosh" with "cos"

   (Thanks to Farid Bouttout for correcting this error.)

6. Page 68, Integral 2.110.7: The evaluation is incorrect. When corrected, in variables consistent with the other integrals in this section, we have 7/2022

$$\int x^b(a+bx^k)^m\,\mathrm{d}x = \frac{b^m}{k}\sum_{i=0}^{m}\frac{(-1)^i m!\Gamma\left(\frac{b+1}{k}\right)\left(x^k+\frac{a}{b}\right)^{m-i}}{(m-i)!\Gamma\left(\frac{b+1}{k}+i+1\right)}x^{b+1+ki}$$

7. Page 71, Integral 2.124: The first evaluation is incorrect; replace $x\sqrt{\frac{ab}{a}}$ with $x\sqrt{\frac{b}{a}}$.

   (Thanks to Leland Langston for correcting this error.)

8. Page 75, Integral 2.144.1: The auxiliary functions are incorrect. They should have been 7/2022

$$P_k = \frac{1}{2}\ln\left(x^2 - 2x\cos\frac{2k\pi}{n}+1\right), \qquad Q_k = \arctan\left(\frac{x-\cos\frac{2k\pi}{n}}{\sin\frac{2k\pi}{n}}\right)$$

   Additionally, the reference should be changed to be INT31.

9. Page 75, Integral 2.144.2: The integral evaluation is incorrect. It should have been 7/2022

$$\int\frac{\mathrm{d}x}{1-x^n} = -\frac{1}{n}\ln(1-x) - \frac{2}{n}\sum_{k=1}^{\frac{n-1}{2}}P_k\cos\frac{2k\pi}{n} + \frac{2}{n}\sum_{k=1}^{\frac{n-1}{2}}Q_k\sin\frac{2k\pi}{n}$$

   Additionally, the reference should be changed to be INT31.

10. Page 79, Integral 2.172: The evaluation is improved by replacing $\left(\frac{b+2cx}{\sqrt{-\Delta}}\right)$ with $\left(\frac{\sqrt{-\Delta}}{b+2cx}\right)$ for the case $\Delta<0$. Since $\operatorname{arctanh} z$ is equal to $\operatorname{arctanh}\frac{1}{z}$ plus a constant, the evaluation is structually the same. However, complex constants are avoided since the arctanh argument does not exceed one.

    (Thanks to Leland Langston for this improvement.)

11. Page 90, Integral 2.245.2: The differential is currently "dz", which is is incorrect; it should be "dx". 7/2022

12. Page 105, Constraints in 2.292-2 and 2.292-3.

    The constraints for integrals 2.292-2 and 2.292-3 should be written as

2.292-2	$z=\frac{\sqrt{x(1-x)(1-k^2x)}}{1-x}$
2.292-3	$z=\frac{\sqrt{x(1-x)(1-k^2x)}}{1-k^2x}$

    The current statements are incorrect since they made simplifications such as $\frac{\sqrt{y}}{y}=\frac{1}{\sqrt{y}}$ which is only sometimes true, because of branch cuts.

    (Thanks to Sam Blake for correcting these errors.)

13. Page 109, Integral 2.33.16: replace 'exp" with "erf".

    (Thanks to Aaron Hendrickson for correcting this error.)

14. Page 184, line 7: Disregard the spurious text “ndexsquare roots”

15. Page 184, integral 2.581.1

    To correct this integral, in the first line of the evaluation change
    $[m+n-2(m+r-1)k^2]$ to
    $[(m+n-2)+(m+r-1)k^2]$.

    (Thanks to Peng Zhang for correcting this error.)

16. Page 194, integral 2.584.63.

    The last term in the evaluation is incorrect; the power of Δ in the denominator should be Δ^3 (not Δ).
    That is, the last term should be $\dfrac{k^2(2k^2-1)\sin^2 x - 3k^2 + 2}{3k'^2\Delta^3}\sin x\cos x$

    (Thanks to Michael James Ungs for correcting this error.)

17. Page 218, Integral 2.637.4: replace the first $\frac{3}{2}$ (in the parentheses) with $-\frac{3}{2}$ and replace $\frac{1}{54}$ with $-\frac{1}{54}$. 7/2022

18. Page 224, Integral 2.647.6: replace $\frac{\pi}{2}$ with $\frac{x}{2}$.

19. Page 255, expressions in 3.112: the two expansions are each missing a plus sign.
    They should be (the additional plus signs are shown boxed):

$$g_n(x) = b_0x^{2n-2}\boxed{+}b_1x^{2n-4} + \cdots + b_{n-1},$$
$$h_n(x) = a_0x^{n}\boxed{+}a_1x^{n-1} + \cdots + a_n,$$

    (Thanks to Mo Li for correcting these errors.)

20. Page 255, Integrals in 3.112

    3.112.1 The correct evaluation is $(-1)^{n+1}\dfrac{\pi i}{a_0}\dfrac{M_n}{\Delta_n}$
    (The first term was missing.)

    Additionally, add the following text after the two determinants:

    The matrices for $n = 5$ are:

$$\Delta_5 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_0 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix}, \qquad M_5 = \begin{vmatrix} b_0 & b_1 & b_2 & b_3 & b_4 \\ a_0 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_0 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix}$$

    3.112.3 The correct evaluation is $\dfrac{-a_2b_0 + a_0b_1 - \dfrac{a_0a_1b_2}{a_3}}{a_o(a_0a_3 - a_1a_2)}$
    (There were two operators missing.)

    (Thanks to Hongjun Xiang for correcting these errors.)

21. Page 264, Integral 3.137.7: for the last term, replace "$(a-p)\,F(\mu,q)$" with "$(a-r)\,F(\mu,q)$".

    (Thanks to Elliot Blackstone for correcting this error.)

22. Page 277, Integral 3.145.3(2): change the first integral from $\int_u^a$ to $\int_u^\alpha$.

    (Thanks to Dominik Beck for correcting this error.)

23. Page 326, Integral 3.248.5

    When integral 3.248.5 in the 6th edition was found to be incorrect the entry was removed; neither the 7th or 8th edition had an entry for 3.248.5. The correct evaluation was determined in the paper AR.

    The integral (6th edition, page 321) is

    $$\int_0^\infty \frac{\mathrm{d}x}{(1+x^2)^{3/2}} \frac{1}{\sqrt{\phi(x)+\sqrt{\phi(x)}}} = \frac{\pi}{2\sqrt{6}} \qquad \boxed{\text{incorrect}}$$

    which is incorrect. It should have been

    $$\int_0^\infty \frac{\mathrm{d}x}{(1+x^2)^{3/2}} \frac{1}{\sqrt{\phi(x)+\sqrt{\phi(x)}}} = \frac{\sqrt{3}-1}{\sqrt{2}}\Pi\left(\frac{\pi}{2},k,3^{-1/2}\right) - \frac{1}{\sqrt{2}}F\left(\alpha,3^{-1/2}\right)$$

    with $\phi(x)=1+\frac{4}{3}\left(\frac{x}{1+x^2}\right)^2$, $k=2-\sqrt{3}$, and $\alpha=\arcsin\sqrt{k}$ and the reference AR.

24. Page 326, add new Integral 3.248.7

    In the search for the correct evaluation of 3.248.5 (see note above), a small variation of the integral was found (in an unpublished paper by Juan Arias de Reyna, Petr Blaschke, and Victor H. Moll). This, perhaps, explains the original typographic error in 3.248.5.

    $$\int_0^\infty \frac{\mathrm{d}x}{(1+x^2)^{3/2}} \frac{1}{\sqrt{\phi(x)+\sqrt{\phi(x)^3}}} = \frac{\pi}{2\sqrt{6}}$$

    with $\phi(x)=1+\frac{4}{3}\left(\frac{x}{1+x^2}\right)^2$.

25. Page 329, Integral 3.252.11: replace $\left(\beta^2-1\right)$ with $\left(1-\beta^2\right)$.

26. Page 336, Integral 3.311.1: add the constraint $\operatorname{Re} p > 0$; add the reference VE2015

27. Page 336, Integral 3.311.5: replace $\operatorname{Re}\nu < 1$ with $\operatorname{Re}\nu < 0$; add the reference VE2015

28. Page 337, Integral 3.312.1: replace $\operatorname{Re}\nu > 0$ with $\operatorname{Re}\nu > 1$; add the reference VE2015

29. Page 338, Integral 3.318.2: replace $\sqrt{\pi e^{\cdots}}$ with $\sqrt{\pi}e^{\cdots}$; add the reference VE2015

30. Page 338, Integral 3.321.3: replace $\frac{\sqrt{\pi}}{2q}$ $[q > 0]$ with $\frac{\sqrt{\pi}}{2\sqrt{q^2}}$ $[\operatorname{Re} q^2 > 0]$; add the reference VE2015

31. Page 339, Integral 3.322.1: remove $\operatorname{Re}\beta > 0, \quad u > 0$; add the reference VE2015

32. Page 339, Integral 3.323.2: replace $\dfrac{\sqrt{\pi}}{p}$ with $\dfrac{\sqrt{\pi}}{\sqrt{p^2}}$; add the reference VE2015

33. Page 339, Integral 3.323.3: add the constraint $[\operatorname{Re} a > 0]$; add the reference VE2015

34. Page 339, Integral 3.323.4: add the constraint $[\operatorname{Re}\beta^2 > 0, \quad \operatorname{Re}\gamma^2 > 0]$; add the reference VE2015

35. Page 344, Integral 3.354.5: replace $\dfrac{\pi}{a}$ with $\dfrac{\pi}{|a|}$; add the reference VE2015

36. Page 350, Integral 3.383.5

    The evaluation of the integral is incorrect. The correct evaluation is

$$= \frac{\pi^2}{p^q\,\Gamma(\nu)\,\sin[\pi(q-\nu)]}\left[\left(\frac{p}{a}\right)^{\nu}\frac{L_{-\nu}^{\nu-q}\left(\frac{p}{a}\right)}{\sin(\pi\nu)\,\Gamma(1-q)} - \left(\frac{p}{a}\right)^{q}\frac{L_{-q}^{q-\nu}\left(\frac{p}{a}\right)}{\sin(\pi q)\,\Gamma(1-\nu)}\right]$$

    (Thanks to Mohammad S. Alhassoun for correcting this error.)

37. Page 351, Integral 3.385

    (a) The evaluation of the integral is incorrect; the term
        $\Phi_1(\nu, \varrho, \lambda+\nu, -\mu, b)$ should be
        $\Phi_1(\nu, \varrho, \lambda+\nu, b, -\mu)$
    (b) The reference is incorrect. It is now "ET 1 39(24)", it should be "ET 1 139(24)".

    (Thanks to Travis Porco for correcting these errors.)

38. Page 358, Integral 3.416.3: replace 2^{2^n} with 2^{2n}; add the reference VE2015

39. Page 358, Integral 3.417.1: replace $\dfrac{\pi}{2ab}\ln\left(\dfrac{b}{a}\right) \quad [ab > 0]$ with $\dfrac{\pi}{2|ab|}\ln\left(\left|\dfrac{b}{a}\right|\right) \quad [a \neq 0, \quad b \neq 0]$; add the reference VE2015

40. Page 361, Integral 3.426.2

    The numerator of the integrand is incorrect; the term "$(e^x - ae^{-x})$" should be "$(e^x + ae^{-x})$".

41. Page 369, Integral 3.462.22: replace "$K_1(ab)$" with "$K_2(ab)$".

    (Thanks to Peter Brown for correcting this error.)

42. Page 369, Integral 3.462.25: replace $\operatorname{Re} b > 0$ with $\operatorname{Re} p > 0$; add the reference VE2015

43. Page 369, Integral 3.466.1: expand the evaluation with

$$\begin{aligned} &[1-\Phi(b\mu)]\,\frac{\pi}{2b}e^{b^2\mu^2} \qquad [\operatorname{Re} b > 0, \quad |\arg\mu| < \tfrac{\pi}{4}] \\ -&[1+\Phi(b\mu)]\,\frac{\pi}{2b}e^{b^2\mu^2} \qquad [\operatorname{Re} b < 0, \quad |\arg\mu| < \tfrac{\pi}{4}] \end{aligned}$$

    and add the reference VE2015

44. Page 369, Integral 3.468.2: the u should have been a μ; but it better to write the integral using a single parameter

$$\int_0^\infty \frac{xe^{-\beta^2x^2}\,dx}{\sqrt{1+x^2}} = \frac{\sqrt{\pi}}{2\beta}e^{\beta^2}\left[1-\Phi(\beta)\right] \qquad [\operatorname{Re}\beta^2 > 0]$$

45. Page 374, Integral 3.512.2
    (a) Replace $\frac{1}{2}\,\mathrm{B}\left(\frac{\mu+1}{2}, \frac{\nu-\boxed{1}}{2}\right)$ with $\frac{1}{2}\,\mathrm{B}\left(\frac{\mu+1}{2}, \frac{\nu-\mu}{2}\right)$
    (b) Replace the constraints with $[\operatorname{Re}(\nu) > \operatorname{Re}(\mu) > -1]$

    (Thanks to Shotaro Yamazoe for correcting this error.)

46. Page 382, Integral 3.527.13: in the denominator of the integrand replace "$\cosh^2 x$" with "$\sinh^2 x$". 7/2022

47. Page 384, Integral 3.536.2: in the denominator of the integrand replace "$\cosh^2 x$" with "$\cosh x^2$".

48. Page 419, Integral 3.691.2: replace $S\left(\sqrt{a}\right)$ with $S\left(\sqrt{\frac{2a}{\pi}}\right)$; add the reference VE2015

49. Page 419, Integral 3.691.3: replace $C\left(\sqrt{a}\right)$ with $C\left(\sqrt{\frac{2a}{\pi}}\right)$; add the reference VE2015

50. Page 419, for Integrals 3.691.4, 3.691.6, 3.691.8, and 3.691.9: replace $C\left(\frac{b}{\sqrt{a}}\right)$ with $C\left(b\sqrt{\frac{2}{a\pi}}\right)$ and replace $S\left(\frac{b}{\sqrt{a}}\right)$ with $S\left(b\sqrt{\frac{2}{a\pi}}\right)$; add the reference VE2015

    for .8 and .9 check what the replacement should be

51. Page 429, Integral 3.725.3: the evaluation of the integral should be changed to the following

$$\begin{array}{rl}
\gamma_1 & [\operatorname{Re}\beta > 0, \quad 0 < a < b] \\
\gamma_1 & [\operatorname{Re}\beta > 0, \quad a < 0 < b] \\
-\gamma_1 & [\operatorname{Re}\beta < 0, \quad b < a < 0] \\
\gamma_2 & [\operatorname{Re}\beta < 0, \quad 0 < a < b] \\
\gamma_2 & [\operatorname{Re}\beta < 0, \quad a < 0 < b] \\
-\gamma_2 & [\operatorname{Re}\beta > 0, \quad b < a < 0] \\
\gamma_3 & [\operatorname{Re}\beta > 0, \quad 0 < b < a] \\
\gamma_3 & [\operatorname{Re}\beta > 0, \quad b < 0 < a] \\
-\gamma_3 & [\operatorname{Re}\beta < 0, \quad a < b < 0] \\
\gamma_4 & [\operatorname{Re}\beta < 0, \quad 0 < b < a] \\
\gamma_4 & [\operatorname{Re}\beta < 0, \quad b < 0 < a] \\
-\gamma_4 & [\operatorname{Re}\beta > 0, \quad a < b < 0]
\end{array}$$

where

$$\begin{aligned}
\gamma_1 &= \quad \frac{\pi}{2\beta^2} e^{-b\beta} \sinh(a\beta) \\
\gamma_2 &= -\frac{\pi}{2\beta^2} e^{b\beta} \sinh(a\beta) \\
\gamma_3 &= -\frac{\pi}{2\beta^2} e^{-a\beta} \cosh(b\beta) + \frac{\pi}{2\beta^2} \\
\gamma_4 &= -\frac{\pi}{2\beta^2} e^{a\beta} \cosh(b\beta) + \frac{\pi}{2\beta^2}
\end{aligned}$$

and add the reference VE2015

52. Page 439, Integral 3.755.1: add the constraint $\operatorname{Re} b > 0$; add the reference VE2015

53. Page 447, Integral 3.772.5: replace "ET I 12(4)" with "read ET I 12(14)"; add the reference VE2015

54. Page 489, Integrals 3.891.1 and 3.891.2.

    In each case the results are correct, but only when m and n are non-negative integers. The result when m and n can be any integers are:

$$\text{3.891.1} \quad \int_0^{2\pi} e^{imx} \sin nx \, dx = \begin{cases} 0 & |m| \neq |n| \text{ or } m = n = 0 \\ \pi i & m = n \neq 0 \\ -\pi i & m = -n \neq 0 \end{cases}$$

$$\text{3.891.2} \quad \int_0^{2\pi} e^{imx} \cos nx \, dx = \begin{cases} 0 & |m| \neq |n| \\ \pi & |m| = n \neq 0 \\ 2\pi & m = n = 0 \end{cases}$$

    (Thanks to Guillem Blanco for correcting these errors.)

55. Page 495, Integral 3.914.6:

    (a) change the reference from "ET I 175(35)" to "ET I 75(35)" to
    (b) add the contraints $[\operatorname{Re} \beta > 0, \quad \operatorname{Re} \gamma > 0]$

    (Thanks to Claudio Severi and Howard Haber for correcting these errors.)

56. Page 495, Integral 3.914.9: add the contraints $[\operatorname{Re} \beta > 0, \quad \operatorname{Re} \gamma > 0]$

    (Thanks to Howard Haber for correcting this error.)

57. Page 525, Integral 4.124.2: delete the current integral and replace with the following

$$\int_0^u \frac{\cos px \cosh(q\sqrt{u^2 - x^2})}{\sqrt{u^2 - x^2}} \, dx = \frac{\pi}{2} J_0 \left(u\sqrt{p^2 - q^2} \right) \qquad [u > 0]$$

    (Thanks to Mark Coffey for correcting this error.)

58. Page 535, Integral 4.22.8: delete the current evaluation and replace it with the following 7/2022

$$\int_0^\infty \ln(1+ax)x^b e^{-x}\,dx = -\sum_{k=0}^{b} \frac{b!}{(b-k)!}\frac{1}{(-a)^{b-k}} \times \left[e^{1/a}\mathrm{Ei}\left(-\frac{1}{a}\right) - \sum_{j=1}^{b-k}(j-1)!\,(-a)^j \right]$$
$$[a>0, \quad b>0 \text{ an integer}]$$

59. Page 535, Integral 4.224.12: remove the evaluation for $a^2 \geq 1$; remove the reference

(Thanks to Martin Kreh and Richard Hunt for correcting these errors.)

60. Page 535, Integral 4.224.12 (1)

The integrand has the exponent of "2" in the wrong place. The evaluation is correct. That is, replace the current entry with

$$\int_0^\pi \ln\left(1+a\cos x\right)^2\,\mathrm{d}x = \begin{cases} 2\pi\ln\left(\dfrac{1+\sqrt{1-a^2}}{2}\right) & \text{for } a^2 \leq 1 \\ 2\pi\ln\left(\dfrac{|a|}{2}\right) & \text{for } a^2 \geq 1 \end{cases}$$

And add the reference "BI (330)(1)".

(Thanks to Martin Kreh and Richard Hunt for correcting these errors.)

61. Page 539, Integral 4.231.19

The correct evaluation of this integral is (the "2" should be a "12")

$$\int_0^1 \frac{x\log x}{1+x}\,\mathrm{d}x = -1 + \frac{\pi^2}{12}$$

(Thanks to Kendall Richards for correcting this error.)

62. Page 539, Integral 4.232.1

For consistency with other entries, the term "log" appearing in the output should be "ln".

63. Page 558, Integral 4.283.9

The correct evaluation of this integral is as follows (the q was missing from the integrand and the evaluation had switched a and q)

$$\int_0^1 \left[x^q + \frac{1}{a\ln x - 1}\right]\frac{\mathrm{d}x}{x\ln x} = \ln\left(\frac{q}{a}\right) + \boldsymbol{C}$$

64. Page 572, Integral 4.319.1, replace the current evaluation with

$$-\frac{\pi}{2}\left(2a + \ln\left(\frac{\Gamma^2\left(a+1\right)}{2\pi a^{2a+1}}\right)\right) \qquad \mathrm{Re}(a) > 0 \qquad \text{REY3 (16)}$$

(Thanks to Robert Reynolds correcting this error.)

65. Page 572, Integral 4.319.2, replace the current evaluation with

$$-\pi\left(a+\ln\left(\frac{\Gamma\left(a+\frac{1}{2}\right)}{a^a\sqrt{2\pi}}\right)\right) \qquad \mathrm{Re}(a)>0 \qquad \text{REY3 (15)}$$

(Thanks to Robert Reynolds correcting this error.)

66. Page 580, Integral 4.358.2: replace $\zeta(2,\nu-1)$ with $\zeta(2,\nu)$; add the reference VE2015

67. Page 634, Integral 5.54.3: for the evaluation, replace $\frac{x^4}{4}$ with $\frac{x^2}{4}$
(Thanks to Adriana Brancaccio and Brady Metherall for (independently) correcting this error.)

68. Page 634, Integral 5.54.3: add the reference WA 134(10)
(Thanks to Brady Metherall for correcting this error.)

69. Page 654, Integral 6.282.2: add the constraint $\mathrm{Re}\,\mu>0$; add the reference VE2015

70. Page 654, Integral 6.283.1: replace "$\mathrm{Re}\,\alpha>0$" with "$\mathrm{Re}\,\beta<0$"; add the reference VE2015

71. Page 654, Integral 6.285.1: expand the evaluation by replacing

$$\frac{\arctan\mu}{\sqrt{\pi}\mu} \qquad [\mathrm{Re}\,\mu>0]$$

with

$$\frac{\arctan\sqrt{\mu^2}}{\sqrt{\pi}\sqrt{\mu^2}} \qquad [\mathrm{Re}\,\mu^2>0]$$

Add the reference VE2015

72. Page 654, Integral 6.285.2: change sign of result by replacing $-\dfrac{1}{2ai\sqrt{\pi}}$ with $\dfrac{1}{2ai\sqrt{\pi}}$; add the reference VE2015

73. Page 655, Integral 6.291: replace $\frac{\mu}{a}$ with $\frac{\mu}{4}$; add the reference VE2015

74. Page 655, Integral 6.295.2: replace $-\frac{1}{\mu^2}$ with $-\frac{1}{\mu}$; add the reference VE2015

75. Page 656, Integral 6.296: replace "$a>0$" with "a real"; add the reference VE2015

76. Page 656, Integral 6.297.1: add the constraint $\mathrm{Re}(\gamma^2-\mu)<0$; add the reference VE2015

77. Page 656, Integral 6.297.2: replace "$a>0, b>0, \mathrm{Re}\,\mu>0$" with "$b>0, \mathrm{Re}(\mu^2-a^2)>0$"; add the reference VE2015

78. Page 656, Integral 6.297.3: remove $a>0$; add the reference VE2015

79. Page 656, Integral 6.298: replace the constraint with "$[b>0, \quad \mathrm{Re}\,\mu>0, \quad \mathrm{Re}(\mu-a^2)>0]$"; add the reference VE2015

80. Page 656, Integral 6.299: replace $K_\nu(a^2)$ with $K_\nu(\frac{1}{2}a^2)$; add the reference VE2015

81. Page 656, Integral 6.311: generalize the evaluation to be

$$\frac{1}{b}\left(1-e^{-b^2/4a^2}\right) \qquad [a>0, \quad b\neq 0]$$
$$\frac{1}{b}\left(1+e^{-b^2/4a^2}\right) \qquad [a<0, \quad b\neq 0]$$

and add the reference VE2015

82. Page 656, Integral 6.312: expand the evaluation, and correct the constraint, with

$$\frac{1}{4\sqrt{2\pi b}}\left[\ln\left(\frac{b+a^2+a\sqrt{2b}}{b+a^2-a\sqrt{2b}}\right)+2\arctan\left(\frac{a\sqrt{2b}}{b-a^2}\right)\right] \qquad [a>0, \quad b>0, \quad a<\sqrt{b}]$$
$$\frac{1}{4\sqrt{2\pi b}}\left[\ln\left(\frac{b+a^2+a\sqrt{2b}}{b+a^2-a\sqrt{2b}}\right)+2\arctan\left(\frac{a\sqrt{2b}}{b-a^2}\right)+2\pi\right] \qquad [a>0, \quad b>0, \quad a>\sqrt{b}]$$

and add the reference VE2015

83. Page 657, Integral 6.314.1: the integral and its solution should be replaced by

$$\int_0^\infty \sin(bx)\Phi\left(\sqrt{\frac{a}{x}}\right)\,\mathrm{d}x = \frac{1}{b}\left(1-\cos\left(\sqrt{2ab}\right)\exp^{-\sqrt{2ab}}\right) \qquad [\operatorname{Re} a>0, \quad \operatorname{Re} b>0]$$

and add the reference VE2015

84. Page 657, Integral 6.314.2: the integral and its solution should be replaced by

$$\int_0^\infty \cos(bx)\Phi\left(\sqrt{\frac{a}{x}}\right)\,\mathrm{d}x = \frac{1}{b}\sin\left(\sqrt{2ab}\right)\exp^{-\sqrt{2ab}} \qquad [\operatorname{Re} a>0, \quad \operatorname{Re} b>0]$$

and add the reference VE2015

85. Page 657, Integral 6.315.3: replace $b>0$ with $b\neq 0$; add the reference VE2015

86. Page 657, Integral 6.315.4: replace $\operatorname{Ei}\left(\frac{p}{4a^2}\right)$ with $\operatorname{Ei}\left(-\frac{p}{4a^2}\right)$ and replace $p>0$ with $p\neq 0$; add the reference VE2015

87. Page 657, Integral 6.315.5: expand the evaluation, and correct the constraint, with

$$\frac{1}{2\sqrt{2\pi b}}\left[\ln\left(\frac{b+a\sqrt{2b}+a^2}{b-a\sqrt{2b}+a^2}\right)+2\arctan\left(\frac{a\sqrt{2b}}{b-a^2}\right)\right] \qquad [a>0, \quad b>0, \quad a<\sqrt{b}]$$
$$\frac{1}{2\sqrt{2\pi b}}\left[\ln\left(\frac{b+a\sqrt{2b}+a^2}{b-a\sqrt{2b}+a^2}\right)+2\arctan\left(\frac{a\sqrt{2b}}{b-a^2}\right)+2\pi\right] \qquad [a>0, \quad b>0, \quad a>\sqrt{b}]$$

and add the reference VE2015

88. Page 657, Integral 6.317: expand the evaluation, and correct the constraint, with

$$\frac{i}{a}\frac{\sqrt{\pi}}{2}e^{-\frac{b^2}{4a^2}} \qquad [\operatorname{Re} a^2 > 0, \quad b > 0]$$
$$-\frac{i}{a}\frac{\sqrt{\pi}}{2}e^{-\frac{b^2}{4a^2}} \qquad [\operatorname{Re} a^2 > 0, \quad b < 0]$$

and add the reference VE2015

89. Page 657, Integral 6.318: the correct evaluation is

$$\frac{1}{2p}\left(e^{-p^2} - 1\right) + \frac{\sqrt{\pi}}{2}\Phi(p) \qquad [\operatorname{Re} p > 0]$$

and add the reference VE2015

90. Page 668, Integral 6.511.7: generalize the result to

$$\int_0^a J_1(xy)\,\mathrm{d}x = \frac{1}{y}\left[1 - J_0(ay)\right] \qquad [a > 0, \quad y \neq 0]$$

and add the reference VE2015

91. Page 668, Integral 6.511.9: remove the constraint; add the reference VE2015

92. Page 669, Integral 6.512.9: replace $b > 0$ with $b \neq 0$; add the reference VE2015

93. Page 669, Integral 6.512.10: replace the constraint with $[a > 0, \quad b \neq 0, \quad a > |b|]$; add the reference VE2015

94. Page 671, Integral 6.516.1: include the additional evaluation

$$-\frac{1}{b}J_\nu\left(\frac{a^2}{4b}\right) \qquad [a > 0, \quad b < 0, \quad \operatorname{Re}\nu > -\tfrac{1}{2}]$$

and add the reference VE2015

95. Page 671, Integral 6.516.4: add the constraint $\operatorname{Re}\nu > -\frac{1}{2}$; add the reference VE2015

96. Page 673, Integral 6.521.2: replace the constraint with $[\operatorname{Re}(a \pm ib) > 0, \quad \operatorname{Re}\nu > -1]$; add the reference VE2015

97. Page 673, Integral 6.521.7: remove $b > 0$; add the reference VE2015

98. Page 673, Integral 6.521.8: replace the constraint with $[a > |b| \geq 0]$; add the reference VE2015

99. Page 673, Integral 6.521.9: replace the constraint with $[a > |b| \geq 0]$; add the reference VE2015

100. Page 673, Integral 6.521.12: remove $b > 0$; add the reference VE2015

101. Page 673, Integral 6.521.13: replace the constraint with $[a > 0]$; add the reference VE2015

102. Page 673, Integral 6.521.14: replace the constraint with $[a > |b| \geq 0]$; add the reference VE2015

103. Page 673, Integral 6.521.15: replace the constraint with $[a > |b| \geq 0]$; add the reference VE2015

104. Page 674, Integral 6.522.4: in the first constraint remove $c > 0$; in the second constraint remove $a > 0$; add the reference VE2015

105. Page 674, Integral 6.522.5: remove the constraint $c > 0$ (in 2 places); add the reference VE2015

106. Page 676, Integral 6.524.2: in the evaluation replace "a" with "$|a|$"; replace the constraint with $[a \neq 0, \quad b > 0]$; add the reference VE2015

107. Page 676, Integral 6.525.1: replace the first constraint with $[\operatorname{Re} b > |\operatorname{Im} a|]$; replace the second constraint with $[\operatorname{Re} a > |\operatorname{Im} b|]$; add the reference VE2015

108. Page 676, Integral 6.525.2: remove the constaint $c > 0$; add the reference VE2015

109. Page 676, Integral 6.525.3: replace $K_0(bx)$ with $K_1(bx)$; replace the constraints with $[\operatorname{Re} b > 0]$; add the reference VE2015

110. Page 676, Integral 6.526.1: replace "$(2a)^{-1}$" with "$(2|a|)^{-1}$"; replace the constraints with $[a \neq 0, \quad b \geq 0, \quad \operatorname{Re} \nu > -1]$; add the reference VE2015

111. Page 679, Integral 6.532.4: expand the evaluation with

$$\begin{array}{rl} K_0(ak) & \text{if } [a > 0, \operatorname{Re} k > 0] \text{ or } [a < 0, \operatorname{Re} k < 0] \\ K_0(-ak) & \text{if } [a > 0, \operatorname{Re} k < 0] \text{ or } [a < 0, \operatorname{Re} k > 0] \end{array}$$

and add the reference VE2015

112. Page 679, Integral 6.532.5: expand the evaluation with

$$\begin{array}{rl} -\dfrac{K_0(ak)}{k} & [a > 0, \quad \operatorname{Re} k > 0] \\ \dfrac{K_0(-ak)}{k} & [a > 0, \quad \operatorname{Re} k < 0] \end{array}$$

and add the reference VE2015

113. Page 679, Integral 6.532.6: expand the evaluation with

$$\begin{array}{rl} \dfrac{\pi}{2k}\left[I_0(-ak) - \mathbf{L}_0(-ak)\right] & [a < 0, \quad \operatorname{Re} k > 0] \\ -\dfrac{\pi}{2k}\left[I_0(-ak) - \mathbf{L}_0(-ak)\right] & [a > 0, \quad \operatorname{Re} k < 0] \\ -\dfrac{\pi}{2k}\left[I_0(ak) - \mathbf{L}_0(ak)\right] & [a < 0, \quad \operatorname{Re} k < 0] \end{array}$$

and add the reference VE2015

114. Page 697, Integral 6.533.3: expand the evaluation with

$$\begin{array}{rl} -\frac{b}{4}\left[1+2\ln\left(\left|\frac{a}{b}\right|\right)\right] & \text{if } (a+b<0) \text{ and } ([0<b<a] \text{ or } [a<b<0] \text{ or } [a<0<b]) \\ -\frac{b}{4}\left[1+2\ln\left(\left|\frac{a}{b}\right|\right)\right] & [a+b>0, \quad b<0<a] \\ -\frac{a^2}{4b} & \text{if } (a+b>0) \text{ and } ([0<a<b] \text{ or } [b<a<0] \text{ or } [a<0<b]) \\ -\frac{a^2}{4b} & [a+b<0, \quad b<0<a] \end{array}$$

and add the reference VE2015

115. Page 683, Integral 6.554.1: expand the evaluation with

$$\begin{array}{rl} y^{-1}e^{ay} & [y>0, \quad \operatorname{Re} a<0] \\ -y^{-1}e^{ay} & [y<0, \quad \operatorname{Re} a>0] \\ -y^{-1}e^{-ay} & [y<0, \quad \operatorname{Re} a<0] \end{array}$$

and add the reference VE2015

116. Page 687, Integral 6.566.2: the evaluation is also valid for the constraints $[a<0, \operatorname{Re} b<0, -1<\operatorname{Re}\nu<\frac{3}{2}]$; add the reference ET II 23(12)

117. Page 687, Integral 6.566.3: expand the evaluation with

$$\begin{array}{rl} \frac{\pi^2 b^{\nu-1}}{4\cos\nu\pi}\left[\mathbf{H}_{-\nu}(ab)-Y_{-\nu}(ab)\right] & [a>0, \quad \operatorname{Re} b>0, \quad \operatorname{Re}\nu>-\frac{1}{2}] \\ \frac{\pi^2 (-b)^{\nu-1}}{4\cos\nu\pi}\left[\mathbf{H}_{-\nu}(-ab)-Y_{-\nu}(-ab)\right] & [a>0, \quad \operatorname{Re} b<0, \quad \operatorname{Re}\nu>-\frac{1}{2}] \end{array}$$

and add the reference VE2015

118. Page 687, Integral 6.566.4: expand the evaluation with

$$\begin{array}{rl} \frac{\pi^2}{4b^{\nu+1}\cos\nu\pi}\left[\mathbf{H}_{\nu}(ab)-Y_{\nu}(ab)\right] & [a>0, \quad \operatorname{Re} b>0, \quad \operatorname{Re}\nu<\frac{1}{2}] \\ \frac{\pi^2}{4(-b)^{\nu+1}\cos\nu\pi}\left[\mathbf{H}_{\nu}(-ab)-Y_{\nu}(-ab)\right] & [a>0, \quad \operatorname{Re} b<0, \quad \operatorname{Re}\nu<\frac{1}{2}] \end{array}$$

and add the reference VE2015

119. Page 687, 6.566.5: expand the evaluation with

$$\begin{aligned} &\frac{\pi}{2b^{\nu+1}}\left[I_\nu(ab)-\mathbf{L}_\nu(ab)\right] && [a>0, \quad \operatorname{Re} b>0, \quad \operatorname{Re}\nu>-\tfrac{5}{2}] \\ -&\frac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab)-\mathbf{L}_\nu(-ab)\right] && [a<0, \quad \operatorname{Re} b>0, \quad \operatorname{Re}\nu>-\tfrac{5}{2}] \\ &\frac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab)-\mathbf{L}_\nu(-ab)\right] && [a>0, \quad \operatorname{Re} b<0, \quad \operatorname{Re}\nu>-\tfrac{5}{2}] \\ &\frac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab)+\mathbf{L}_\nu(-ab)\right] && [a<0, \quad \operatorname{Re} b<0, \quad \operatorname{Re}\nu>-\tfrac{5}{2}] \end{aligned}$$

and add the reference VE2015

120. Page 699, Integral 6.592.7: replace $\sqrt{\pi}\sec(\nu\pi)$ with $\pi\sec\left(\frac{1}{2}\nu\pi\right)$; add the constraint $a \neq 0$; add the reference VE2015

121. Page 702, Integral 6.611.2: replace the constraints with $[\operatorname{Re}(\alpha \pm ib) > 0, \quad |\operatorname{Re}\nu| < 1]$; replace the references with VE2015

122. Page 703, Integral 6.611.9:

   (a) replace the "a" in the constraints with "α"

   (b) replace the correct $\dfrac{1}{\sqrt{\alpha^2-b^2}}\ln\left(\dfrac{\alpha}{b}+\sqrt{\dfrac{\alpha^2}{b^2}-1}\right)$ with the simpler $\dfrac{1}{\sqrt{\alpha^2-b^2}}\operatorname{arccosh}\left(\dfrac{\alpha}{b}\right)$

   (Thanks to Angelo Melino for correcting the error and suggesting the simplification.)

123. Page 704, Integral 6.612.4: the current evaluation is incorrect; the correct evaluation is

$$\frac{1}{b\pi\sqrt{1+\frac{\alpha^2}{4b^2}}}\,\mathrm{K}\left(\frac{1}{1+\frac{\alpha^2}{4b^2}}\right)$$

(Thanks to the Bogazici Physics seniors of 2018 for correcting this error.)

124. Page 705, Integral 6.613: add the constraint $\operatorname{Re} z \geq 0$; add the reference VE2015

125. Page 714, Integral 6.633.2: replace "$a > 0$" with "a real"; add the reference VE2015

126. Page 718, Integral 6.648: replace $\left(\dfrac{a+be^x}{ae^x+b}\right)$ with $\left(\dfrac{a+be^x}{ae^x+b}\right)^\nu$; add the reference VE2015

127. Page 725, Integral 6.671.4

The term "$+\cot(\nu\pi)$" is incorrect and should have been "$\cot(\nu\pi)$"; that is, there should be a multiplication here and not an addition.

Correcting this, and simplifying the terms results in the following evaluation

$$= -\frac{\sin\left(\frac{\nu\pi}{2}\right)}{\sqrt{b^2-a^2}}\left\{\frac{a^\nu \cot(\nu\pi)}{\left(b+\sqrt{b^2-a^2}\right)^\nu} + \frac{\left(b+\sqrt{b^2-a^2}\right)^\nu}{a^\nu \sin(\nu\pi)}\right\}$$

(Thanks to Junggi Yoon for correcting this error.)

128. Page 725, Integral 6.671.7: add the evaluation of "∞" when $a = b$; add the reference VE2015

129. Page 731, Integrals 6.681.5 and 6.681.6: add the constraint $[n = 0, 1, 2, \ldots]$ for each of these.

(Thanks to Jim Morehead for correcting this error.)

130. Page 732, Integral 6.681.12: replace $\frac{\pi}{2}$ with $\frac{\pi^2}{4}$; add the constraint $a \neq 0$; add the reference VE2015

131. Page 734, Integral 6.686.5: replace the constraints with $[a \neq 0, \quad b \neq 0]$; add the reference VE2015

132. Page 738, Integral 6.699.1 add the evaluation for the case $a = b$

$$\frac{\cos\left((\nu+\lambda)\frac{\pi}{2}\right)}{\sqrt{\pi}(2a)^{\lambda+1}}\frac{\Gamma(\nu+\lambda+1)\Gamma(-\lambda-\frac{1}{2})}{\Gamma(\nu-\lambda)} \qquad [b = a, \quad a \geq 0, \quad \operatorname{Re}\lambda < -\tfrac{1}{2}, \quad \operatorname{Re}(\nu+\lambda) > -2]$$

133. Page 738, Integral 6.699.2 add the evaluation for the case $a = b$

$$\frac{(-1)^{1-\lambda/2}}{\sqrt{\pi}(2a)^{\lambda+1}}\frac{\Gamma(-\lambda-\frac{1}{2})\Gamma(1+\nu+\lambda)}{\Gamma(\nu-\lambda)}\sin(\tfrac{1}{2}\nu\pi) \quad [b = a, \quad a \geq 0, \quad \operatorname{Re}\nu > -\lambda-1, \quad \lambda = -2, -4, \ldots]$$

$$\frac{(-1)^{(3-\lambda)/2}}{\sqrt{\pi}(2a)^{\lambda+1}}\frac{\Gamma(-\lambda-\frac{1}{2})\Gamma(\nu+\lambda+1)}{\Gamma(\nu-\lambda)}\cos(\tfrac{1}{2}\nu\pi) \quad [b = a, \quad a \geq 0, \quad \operatorname{Re}\nu > -\lambda-1, \quad \lambda = -1, -3, \ldots]$$

134. Page 754, 6.772.1: expand the evaluations to be

$$-\frac{1}{a}\left[\ln(2a) + \mathbf{C}\right] \qquad [a > 0]$$

$$\frac{1}{a}\left[\ln(-2a) + \mathbf{C}\right] \qquad [a < 0]$$

and add the reference VE2015

135. Page 754, Integral 6.772.2: expand the evaluations to be

$$-\frac{1}{a}\left[\ln\left(\frac{a}{2}\right) + \mathbf{C}\right] \qquad [a > 0]$$

$$-\frac{1}{a}\left[\ln\left(-\frac{a}{2}\right) + \mathbf{C}\right] \qquad [a < 0]$$

and add the reference VE2015

136. Page 754, Integral 6.772.3: replace $\frac{2}{b}\left(K_0(ab)+\ln a\right)$ with $\frac{2}{b}\left(K_0(|ab|)+\ln|a|\right)$; add the constraints $[a \neq 0, \quad b \neq 0]$; add the reference VE2015

137. Page 754, Integral 6.772.4: expand the evaluations to be

$$\begin{aligned} &\frac{2}{x}\ker(x) && x>0 \\ &\frac{2}{x}\ker(-x) && x<0 \end{aligned}$$

and add the reference VE2015

138. Page 755, Integral 6.784.1: the solution is wrong. The correct solution is

$$\frac{1}{2\sqrt{\pi}}\left(\frac{b}{2}\right)^{\nu}\frac{1}{a^{2\nu+2}}\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(\nu+2\right)}\Phi\left(\nu+\frac{3}{2},\nu+2;-\frac{b^2}{4a^2}\right)$$

In the constraints, replace $b>0$ with $b \neq 0$; add the reference VE2015

139. Page 758, Integral 6.794.9

The current evaluation is

$$\frac{\pi^{3/2}a}{\boxed{2^{5/2}}\sqrt{b}}\exp\left(-b-\frac{a^2}{8b}\right)$$

which is incorrect. The correct evaluation is

$$\frac{\pi^{3/2}a}{2^{7/2}\sqrt{b}}\exp\left(-b-\frac{a^2}{8b}\right)$$

(Thanks to Angelo Melino for correcting this error.)

140. Page 760, Integral 6.812.1: expand the evaluations to be

$$\begin{aligned} &\frac{\pi}{2a}\left[I_1(ab)-\mathbf{L}_1(ab)\right] && [\operatorname{Re} a>0, \quad b>0] \\ &\frac{\pi}{2a}\left[I_1(-ab)-\mathbf{L}_1(-ab)\right] && [\operatorname{Re} a>0, \quad b<0] \\ -&\frac{\pi}{2a}\left[I_1(-ab)-\mathbf{L}_1(-ab)\right] && [\operatorname{Re} a<0, \quad b>0] \\ -&\frac{\pi}{2a}\left[I_1(ab)-\mathbf{L}_1(ab)\right] && [\operatorname{Re} a<0, \quad b<0] \end{aligned}$$

and add the reference VE2015

141. Page 761, Integral 6.812.2: replace $\dfrac{a^2b^2}{2}$ with $\dfrac{a^2b^2}{4}$; add the reference VE2015

142. Page 761, Integral 6.813.4: replace $a>0$ with $a \neq 0$; add the reference VE2015

143. Page 761, Integral 6.813.5: replace $a>0$ with $a \neq 0$; add the reference VE2015

144. Page 770, Integral 6.876.1: replace "$x \operatorname{kei} x\, J_1(ax)$" with "$\operatorname{kei}(x)\, J_1(ax)$"; replace $a>0$ with $a \neq 0$; add the reference VE2015

145. Page 770, Integral 6.876.2: replace "$x \ker x\ J_1(ax)$" with "$\ker(x)\ J_1(ax)$"; replace $a > 0$ with $a \neq 0$; add the reference VE2015

146. Page 779, Integral 7.132.1: replace $\Gamma\left(\lambda + \frac{1}{2}\nu + 1\right)$ with $\Gamma\left(\lambda + \frac{1}{2}\nu + \frac{1}{2}\right)$.

     (Thanks to Bruno Daniel for correcting this error.)

147. Page 799, Integral 7.233: replace $\Gamma(\mu + n)$ with $\Gamma(\mu + n + 1)$

     (Thanks to Ramakrishna Janaswamy for correcting this error.)

148. Page 801, Formula 7.249 2:

     (a) change $\sum_{t=0}^{t-1}$ to $\sum_{r=0}^{t-1}$

     (b) change $[t > n]$ to [any integer $t > n$]

     (Thanks to Matt Majic for correcting this error.)

149. Page 801, Integral 7.251.3: replace $y > 0$ with $y \neq 0$; add the reference VE2015

150. Page 802, Integral 7.251.7: replace $\Gamma\left(\frac{1}{2} + \frac{1}{2}\nu - n\right)$ with $\Gamma\left(\frac{1}{2}\mu + \frac{1}{2}\nu - n\right)$

     (Thanks to Ramakrishna Janaswamy for correcting this error.)

151. Page 810, Integral 7.354.1: replace $J_{2n+1}(x)$ with $J_{2n+1}(z)$.

     (Thanks to Farid Bouttout for correcting this error.)

152. Page 810, Integral 7.355.1: remove the contraint $a > 0$; add the reference VE2015

153. Page 810, Integral 7.355.2: remove the contraint $a > 0$; add the reference VE2015

154. Page 811, Integral 7.374.4: the correct evaluation is $\sqrt{\pi}2^{n-1}\dfrac{(2m+n)!}{m!}(a^2-1)^m a^n$

155. Page 811, Integral 7.374.7: replace $L_n^{n-m}(-2y^2)$ with $L_m^{n-m}(-2y^2)$; remove the constaint $m \leq n$; add the reference VE2015

156. Page 812, Integral 7.376.3: replace $\Gamma\left(\dfrac{\nu+1}{2}\right)$ with $\Gamma\left(\dfrac{\nu}{2} + 1\right)$; add the reference VE2015

157. Page 812, Integral 7.377: replace y^{n-m} with z^{n-m}

     (Thanks to Christophe De Beule for correcting this error.)

158. Page 819, Integral 7.421.1: remove $y > 0$; add the reference VE2015

159. Page 821, Integral 7.512.6

   (a) The evaluation of the integral is incorrect.
   The evaluation should be $= \dfrac{B(\lambda, \beta - \lambda)}{(1-z)^{\alpha}} = \dfrac{\Gamma(\beta-\lambda)\,\Gamma(\lambda)}{\Gamma(\beta)} \dfrac{1}{(1-z)^{\alpha}}$

   (b) The additional constraint $\operatorname{Re}\lambda > 0$ needs to be added

   (Thanks to Gerald Edgar for correcting this error.)

160. Page 821, Integral 7.521.9: replace $(1-z)^{\sigma}$ with $(1-z)^{-\sigma}$
   (Thanks to Gerald Edgar for correcting this error.)

161. Page 822, Integral 7.522.1
   The current evaluation, which is incorrect, is

$$\frac{\Gamma(\delta)\lambda^{-\gamma}}{\Gamma(\alpha)\Gamma(\beta)} E(\alpha; \beta : \gamma; \delta : \lambda)$$

   The correct evaluation, which only differs in the "punctuation" is

$$\frac{\Gamma(\delta)\lambda^{-\gamma}}{\Gamma(\alpha)\Gamma(\beta)} E(\alpha, \beta, \gamma : \delta : \lambda)$$

   (Thanks to Aaron Hendrickson for correcting this error.)

162. Page 825, Integral 7.531.1: add the constraint $c > 0$; add the reference VE2015

163. Page 840, Integral 7.662.4: the solution for $[a < 0, \quad \operatorname{Re} y > 0, \quad \operatorname{Re}\mu > -\frac{1}{2}]$ is the negative of the solution shown. add the reference VE2015

164. Page 851, Integral 7.731.1: replace $\operatorname{Re}^2 a > 0$ with $\operatorname{Re} a^2 > 0$; add the reference VE2015

165. Page 853, Integral 7.751.1: replace the constraint with $[y \neq 0, \quad a \neq 0, \quad n = 1, 3, 5, 7 \ldots]$; add the reference VE2015

166. Page 853, Integral 7.751.2: replace the constraint with $[y \neq 0, \quad a \neq 0]$; add the reference VE2015

167. Page 853, Integral 7.751.3
   The current integrand can be slightly generalized, and the evaluation simplified, as follows:

$$\int_0^{\infty} J_0(xy) D_{\nu}(ax) D_{\nu+1}(x)\, \mathrm{d}x$$
$$= \begin{cases} -\dfrac{1}{y}\left[D_{\nu}\left(\dfrac{y}{a}\right) D_{\nu+1}\left(-\dfrac{y}{a}\right) - \dfrac{\sqrt{\pi}}{\sqrt{2}\Gamma(-\nu)}\right] & [y \neq 0, \quad a > 0] \\ -\dfrac{1}{y}\; D_{\nu}\left(\dfrac{y}{a}\right) D_{\nu+1}\left(-\dfrac{y}{a}\right) & [y \neq 0, \quad a \neq 0, \quad \nu = 0, 1, 2, \ldots] \end{cases}$$

   add the reference VE2015
   This should replace the current value in the 8th edition (which corresponds to the value $a = 1$).

168. Page 853, Integral 7.752.1: replace $y > 0$ with $y \neq 0$; add the reference VE2015

169. Page 853, Integral 7.752.3: replace $y > 0$ with $y \neq 0$; add the reference VE2015

170. Page 853, Integral 7.752.4: replace $y > 0$ with $y \neq 0$; add the reference VE2015

171. Page 853, Integral 7.752.5: replace $y > 0$ with $y \neq 0$; add the reference VE2015

172. Page 854, Integral 7.752.10: replace $y > 0$ with $y \neq 0$; add the reference VE2015

173. Page 854, Integral 7.752.12: replace $y > 0$ with $y \neq 0$; add the reference VE2015

174. Page 856, Integral 7.755.1: replace $y > 0$ with $y \neq 0$; replace $2^{-3/2}$ with $2^{-1/2}$; add the reference VE2015

175. Page 857, Integral 7.771: add $\beta > 0$ to each constraint, replace "ET II 298(22)" with "ET II 398(22)"

176. Page 897, Integral 8.250.5: add the constraint $[\operatorname{Re} p > 0, \quad y > 0]$; add the reference VE2015

177. Page 897, Integral 8.250.8: replace $\Phi\left(-\frac{x^2}{2}\right)$ with $\Phi\left(-\frac{p^2}{2}\right)$; add the reference VE2015

178. Page 897, Integral 8.250.9
    (a) The evaluation is missing a minus sign; the result should be $-\sqrt{\pi}\Phi(a)\Phi(b)$
    (b) add the reference VE2015

179. Page 898, Formula 8.253.1: replace "F_1" with "${}_1F_1$"; add the reference VE2015

180. Page 898, Formula 8.254: replace "$|\arg(-z)|$" with "$|\arg(z)|$".
    (Thanks to Martin Venker for correcting this error.)

181. Page 900, Integral 8.258.5: replace "$1 \quad \arctan(\sqrt{\beta})$" with "$\arctan(\sqrt{\beta})$"; add the reference VE2015

182. Page 909, Formula 8.352.3: replace $\sum_{k=1}^{m}$ with $\sum_{k=1}^{n}$.
    (Thanks to Mariam Mousa Harb for correcting this error.)

183. Page 909, Integral 8.352.7: replace e^{-z} with e^{-x}.
    (Thanks to Mariam Mousa Harb for correcting this error.)

184. Page 912, After 8.361.8, replace "**4.482** 5" with "**4.282** 5" 7/2022
    (Thanks to Allen Stenger for correcting this error.)

185. Page 916, Relation 8.375.1

  (a) The evaluation on the right hand side is incorrect. The correct evaluation is obtained by replacing $\sum_{k=0}^{\lfloor \frac{q-1}{2} \rfloor}$ with $2\sum_{k=0}^{\lfloor \frac{q-1}{2} \rfloor}$
  (b) The comment says (See also **6.362** 5–7) which is incorrect. It should be (See also **6.363** 5–7)

(Thanks to Allen Stenger for correcting these errors.)

186. Page 984, Relation 8.816

The evaluation on the right hand side is incorrect.

The correct evaluation is obtained by replacing "$(-1)^m$" with "$(-i)^m$".

(Thanks to Joseph Gangestad for correcting this error.)

187. Page 985, Relation 8.820.7

For clarity, replace with (this keeps some of the terms and removes others)

$P_\nu(z) = P_{-\nu-1}(z)$

(Thanks to Angelo Melino for suggesting this clarification.)

188. Page 985, Relation 8.822.1

For clarity, replace the constraint with (this keeps some of the terms and removes others)

$\left[\operatorname{Re} z > 0\right]$

(Thanks to Angelo Melino for suggesting this clarification.)

189. Page 997, Relations in 8.922

  (a) (8.922.1) For clarity, change the summation upper limit from ∞ to n
  (b) (8.922.2) For clarity, change the summation upper limit from ∞ to n
  (c) (8.922.1) Add the additional evaluation

$$z^{2n} = \sum_{k=0}^{n} 2^{2n-2k+1}(4n-4k+1)\frac{(2n)!(2n-k+1)!}{k!(4n-2k+2)!}P_{2n-2k}(z)$$

  (d) (8.922.2) Add the additional evaluation

$$z^{2n+1} = \sum_{k=0}^{n} 2^{2n-2k+2}(4n-4k+3)\frac{(2n+1)!(2n-k+2)!}{k!(4n-2k+4)!}P_{2n-2k+1}(z)$$

(Thanks to Patrick Bruno for correcting these errors.)

190. Page 1004, Integral 8.949.7: replace $(1-x^2)^{c\frac{1}{2}}$ with $(1-x^2)^{\frac{1}{2}}$.

(Thanks to Farid Bouttout for correcting this error.)

191. Page 1008, Relation 8.961.1

While correct, the relation is not in it's most general form.

The current

$$P_n^{(\alpha,\alpha)}(-x) = (-1)^n P_n^{(\alpha,\alpha)}(x)$$

should be replaced with

$$P_n^{(\alpha,\beta)}(-x) = (-1)^n P_n^{(\beta,\alpha)}(x)$$

(Thanks to Michal Wierzbicki for this observation.)

192. Page 1034, Integral 9.221: add the constraint $\mathrm{Re}(\mu \pm \lambda) > -\frac{1}{2}$; add the reference VE2015

193. Page 1037, Formula 9.237.1: replace Ψ (representing the confluent hypergeometric function) with ψ (representing the psi function, the logarithmic derivative of the gama function).

(Thanks to Lasse Schmieding for correcting this error.)

194. Page 1038, Formula 9.245.1: replace "x is real" with "$x \geq 0$"; add the reference VE2015

195. Page 1066, Formula 10.618.1: replace "$x_1 = \sqrt{\ldots}$" with "$x_1 = u_3\sqrt{\ldots}$". (Thanks to Michele Cappellari for correcting this error.)

196. Page 1107, line 13, Reference LI: *7/2022*

(a) Author should be C. F. Lindman, not C. E. Lindeman

(b) "Amsterdam 1867" is part of the title, not the place and date of publication (that is, it should be included in the italicized part)

(Thanks to Allen Stengerfor correcting this error.)