

Addendum to the 31st edition of *Standard Mathematical Tables and Formulae*

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NOTES

1 The latest addendum is available from <http://www.mathtable.com/smtf/>.

1 VOTING POWER

See [Section 3.9.2](#), page 278:

Following are tables of the Shapley–Shubik power index (ϕ) and the Banzhaf power index (β) for small games.

Game	ϕ	β
(3; 2, 1, 1, 1)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(3; 2, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(3; 2, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(4; 2, 1, 1, 1, 1)	(5, 2, 2, 2, 2)/15	(5, 2, 2, 2, 2)/15
(4; 2, 1, 1, 1, 1)	(8, 3, 3, 3, 3)/20	(5, 2, 2, 2, 2)/13
(4; 2, 1, 1, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(4; 2, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(4; 2, 2, 1, 1, 1)	(9, 9, 4, 4, 4)/30	(2, 2, 1, 1, 1)/7
(4; 2, 2, 1, 1)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(4; 2, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(4; 2, 2, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(4; 2, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(4; 3, 1, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(7, 1, 1, 1, 1)/11
(4; 3, 1, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(4; 3, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(4; 3, 2, 1, 1)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(4; 3, 2, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(4; 3, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(4; 3, 3, 1)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(5; 2, 1, 1, 1, 1)	(5, 2, 2, 2, 2)/15	(15, 7, 7, 7, 7)/50
(5; 2, 1, 1, 1, 1)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(5; 2, 1, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(5; 2, 2, 1, 1, 1, 1)	(16, 16, 7, 7, 7, 7)/60	(15, 15, 7, 7, 7, 7)/58
(5; 2, 2, 1, 1, 1)	(9, 9, 4, 4, 4)/30	(7, 7, 3, 3, 3)/23
(5; 2, 2, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(5; 2, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(5; 2, 2, 2, 1, 1, 1)	(7, 7, 7, 3, 3, 3)/30	(7, 7, 7, 3, 3, 3)/30
(5; 2, 2, 2, 1, 1)	(8, 8, 8, 3, 3)/30	(7, 7, 7, 3, 3)/27
(5; 2, 2, 2, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(5; 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(5; 2, 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(5; 2, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(5; 3, 1, 1, 1, 1, 1)	(5, 1, 1, 1, 1, 1)/10	(5, 1, 1, 1, 1, 1)/10
(5; 3, 1, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3)/23
(5; 3, 1, 1, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(5; 3, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(5; 3, 2, 1, 1, 1, 1)	(4, 2, 1, 1, 1, 1)/10	(11, 5, 3, 3, 3, 3)/28
(5; 3, 2, 1, 1, 1)	(27, 12, 7, 7, 7)/60	(11, 5, 3, 3, 3)/25
(5; 3, 2, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(5; 3, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(5; 3, 2, 2, 1, 1)	(12, 7, 7, 2, 2)/30	(5, 3, 3, 1, 1)/13
(5; 3, 2, 2, 1)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(5; 3, 2, 2, 2)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(5; 3, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(5; 3, 3, 1, 1, 1, 1)	(9, 9, 4, 4, 4)/30	(2, 2, 1, 1, 1)/7
(5; 3, 3, 1, 1)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(5; 3, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(5; 3, 3, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(5; 3, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(5; 3, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(5; 4, 1, 1, 1, 1, 1, 1)	(10, 1, 1, 1, 1, 1, 1)/15	(15, 1, 1, 1, 1, 1, 1)/20
(5; 4, 1, 1, 1, 1, 1)	(16, 1, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1, 1)/19
(5; 4, 1, 1, 1, 1)	(9, 1, 1, 1, 1)/12	(7, 1, 1, 1, 1)/10
(5; 4, 1, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(5; 4, 2, 1, 1, 1, 1, 1)	(6, 1, 1, 1, 1, 1, 1)/10	(7, 1, 1, 1, 1, 1, 1)/11
(5; 4, 2, 1, 1, 1, 1)	(9, 1, 1, 1, 1, 1)/12	(7, 1, 1, 1, 1, 1)/10
(5; 4, 2, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(5; 4, 2, 2, 1, 1, 1)	(3, 1, 1, 1, 1)/6	(3, 1, 1, 1, 1)/6
(5; 4, 2, 2, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(5; 4, 3, 1, 1, 1, 1)	(3, 1, 1, 1, 1)/6	(3, 1, 1, 1, 1)/6
(5; 4, 3, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(5; 4, 3, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(5; 4, 4, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(6; 2, 1, 1, 1, 1, 1, 1)	(5, 2, 2, 2, 2, 2)/15	(3, 2, 2, 2, 2, 2)/13
(6; 2, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1)/5
(6; 2, 2, 1, 1, 1, 1, 1)	(16, 16, 7, 7, 7, 7)/60	(11, 11, 5, 5, 5, 5)/42
(6; 2, 2, 1, 1, 1, 1)	(7, 7, 2, 2, 2, 2)/20	(2, 2, 1, 1, 1, 1)/7
(6; 2, 2, 1, 1)	(1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1)/4
(6; 2, 2, 2, 1, 1, 1, 1)	(7, 7, 7, 3, 3, 3, 3)/30	(2, 2, 2, 1, 1, 1, 1)/9
(6; 2, 2, 2, 1, 1)	(14, 14, 14, 9, 9)/60	(5, 5, 5, 3, 3, 3)/21
(6; 2, 2, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(6; 2, 2, 2, 2, 1, 1, 1)	(2, 2, 2, 2, 1, 1, 1)/10	(2, 2, 2, 2, 1, 1, 1)/10
(6; 2, 2, 2, 2, 1)	(1, 1, 1, 1, 0)/4	(1, 1, 1, 1, 0)/4
(6; 2, 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1, 0)/5	(1, 1, 1, 1, 1, 0)/5
(6; 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1)/5
(6; 2, 2, 2, 2)	(1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1)/4
(6; 2, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(6; 3, 1, 1, 1, 1, 1, 1)	(5, 1, 1, 1, 1, 1, 1)/10	(8, 3, 3, 3, 3, 3, 3)/23
(6; 3, 1, 1, 1, 1, 1)	(8, 3, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3, 3)/17
(6; 3, 1, 1, 1, 1)	(1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1)/4
(6; 3, 2, 1, 1, 1, 1, 1, 1)	(4, 2, 1, 1, 1, 1, 1, 1)/10	(19, 11, 5, 5, 5, 5, 5)/50
(6; 3, 2, 1, 1, 1, 1, 1)	(6, 3, 1, 1, 1, 1, 1)/12	(4, 3, 1, 1, 1, 1, 1)/10
(6; 3, 2, 1, 1, 1)	(5, 5, 1, 1, 1)/12	(3, 3, 1, 1, 1)/8
(6; 3, 2, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 3, 2, 2, 1, 1, 1, 1, 1)	(21, 12, 12, 5, 5, 5, 5)/60	(19, 11, 11, 5, 5, 5, 5)/56
(6; 3, 2, 2, 1, 1, 1)	(22, 12, 12, 7, 7, 7)/60	(9, 5, 5, 3, 3, 3, 3)/25
(6; 3, 2, 2, 1)	(3, 1, 1, 1, 1)/6	(2, 1, 1, 1, 1)/5
(6; 3, 2, 2, 2, 1, 1, 1)	(9, 5, 5, 5, 3, 3, 3)/30	(9, 5, 5, 5, 3, 3, 3)/30
(6; 3, 2, 2, 2, 1)	(21, 11, 11, 11, 6, 6)/60	(9, 5, 5, 5, 3, 3)/27
(6; 3, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1)/5
(6; 3, 2, 2, 2)	(1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1)/4
(6; 3, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 3, 3, 1, 1, 1, 1, 1, 1)	(3, 3, 1, 1, 1, 1, 1, 1)/10	(8, 8, 3, 3, 3, 3, 3, 3)/28
(6; 3, 3, 1, 1, 1, 1, 1)	(7, 7, 2, 2, 2, 2)/20	(4, 4, 1, 1, 1, 1, 1)/11
(6; 3, 3, 1, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(6; 3, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(6; 3, 3, 2, 1, 1, 1, 1, 1)	(9, 9, 6, 2, 2, 2, 2)/30	(4, 4, 3, 1, 1, 1, 1)/14
(6; 3, 3, 2, 1, 1, 1)	(19, 19, 14, 4, 4, 4)/60	(4, 4, 3, 1, 1, 1)/13
(6; 3, 3, 2, 1)	(2, 2, 1, 1, 1)/6	(2, 2, 1, 1, 1)/6
(6; 3, 3, 2, 2, 1)	(9, 9, 4, 4, 4, 4)/30	(2, 2, 1, 1, 1, 1)/7
(6; 3, 3, 2, 2)	(2, 2, 1, 1, 1)/6	(2, 2, 1, 1, 1)/6
(6; 3, 3, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(6; 3, 3, 3, 1, 1, 1)	(1, 1, 1, 0, 0)/3	(1, 1, 1, 0, 0)/3
(6; 3, 3, 3, 1)	(1, 1, 1, 0, 0)/3	(1, 1, 1, 0, 0)/3
(6; 3, 3, 3, 2)	(1, 1, 1, 0, 0)/3	(1, 1, 1, 0, 0)/3
(6; 3, 3, 3, 3)	(1, 1, 1, 1)/3	(1, 1, 1, 1)/3

Game	ϕ	β
(6; 4, 1, 1, 1, 1, 1, 1)	(10, 1, 1, 1, 1, 1, 1)/15	(13, 2, 2, 2, 2, 2, 2)/23
(6; 4, 1, 1, 1, 1, 1, 1)	(6, 1, 1, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3, 3, 3)/23
(6; 4, 1, 1, 1, 1, 1)	(3, 1, 1, 1, 1)/6	(2, 1, 1, 1, 1)/5
(6; 4, 1, 1, 1)	(1, 1, 1, 1)/3	(1, 1, 1, 1)/3
(6; 4, 2, 1, 1, 1, 1, 1, 1)	(32, 8, 5, 5, 5, 5, 5, 5)/60	(13, 3, 2, 2, 2, 2, 2, 2)/24
(6; 4, 2, 1, 1, 1, 1, 1)	(39, 9, 4, 4, 4, 4, 4)/60	(6, 2, 1, 1, 1, 1, 1)/11
(6; 4, 2, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1, 1)/10
(6; 4, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(6; 4, 2, 2, 1, 1, 1, 1, 1)	(14, 5, 5, 2, 2, 2, 2, 2)/30	(6, 2, 2, 1, 1, 1, 1, 1)/13
(6; 4, 2, 2, 1, 1)	(6, 2, 2, 1, 1, 1)/12	(6, 2, 2, 1, 1, 1)/12
(6; 4, 2, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0, 1)/5
(6; 4, 2, 2, 2, 1)	(3, 1, 1, 1, 0)/6	(3, 1, 1, 1, 0)/6
(6; 4, 2, 2, 2)	(3, 1, 1, 1, 1)/6	(3, 1, 1, 1, 1)/6
(6; 4, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 4, 3, 1, 1, 1, 1, 1, 1)	(4, 2, 1, 1, 1, 1, 1, 1)/10	(11, 5, 3, 3, 3, 3, 3, 3)/28
(6; 4, 3, 1, 1, 1, 1)	(27, 12, 7, 7, 7, 7)/60	(11, 5, 3, 3, 3, 3, 3)/25
(6; 4, 3, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1, 1)/10
(6; 4, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(6; 4, 3, 2, 1, 1, 1)	(12, 7, 7, 2, 2, 2)/30	(5, 3, 3, 1, 1, 1)/13
(6; 4, 3, 2, 1)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(6; 4, 3, 2, 2)	(3, 1, 1, 1, 1)/6	(3, 1, 1, 1, 1)/6
(6; 4, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 4, 3, 3, 1)	(1, 1, 1, 0, 0)/3	(1, 1, 1, 0, 0)/3
(6; 4, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 4, 4, 1, 1, 1, 1, 1)	(9, 9, 4, 4, 4, 4, 4)/30	(2, 2, 1, 1, 1, 1, 1)/7
(6; 4, 4, 1, 1)	(2, 2, 1, 1, 1)/6	(2, 2, 1, 1, 1)/6
(6; 4, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(6; 4, 4, 2, 1)	(1, 1, 1, 0, 0)/3	(1, 1, 1, 0, 0)/3
(6; 4, 4, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 4, 4, 3)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(6; 5, 1, 1, 1, 1)	(25, 1, 1, 1, 1)/30	(31, 1, 1, 1, 1)/36
(6; 5, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(6; 5, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(6; 5, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 2, 1, 1, 1, 1)	(10, 1, 1, 1, 1, 1)/15	(15, 1, 1, 1, 1, 1)/20
(6; 5, 2, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(6; 5, 2, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(6; 5, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 2, 2, 1, 1)	(6, 1, 1, 1, 1)/10	(7, 1, 1, 1, 1)/11
(6; 5, 2, 2, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(6; 5, 2, 2)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(6; 5, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 3, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(7, 1, 1, 1, 1)/11
(6; 5, 3, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(6; 5, 3, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 3, 2, 1)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(6; 5, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 5, 4, 1, 1)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(6; 5, 4, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(6; 5, 4, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(6; 5, 5, 1)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(7; 2, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1)/6
(7; 2, 2, 1, 1, 1, 1)	(3, 3, 1, 1, 1, 1)/10	(5, 5, 3, 3, 3, 3)/22
(7; 2, 2, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(7; 2, 2, 2, 1, 1, 1)	(13, 13, 13, 7, 7, 7)/60	(2, 2, 2, 1, 1, 1)/9
(7; 2, 2, 2, 1, 1)	(6, 6, 6, 1, 1)/20	(3, 3, 3, 1, 1)/11
(7; 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 2, 2, 2, 2, 1, 1)	(13, 13, 13, 13, 4, 4)/60	(5, 5, 5, 5, 2, 2)/24
(7; 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(7; 2, 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1)/6
(7; 2, 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1)/6
(7; 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(7; 2, 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4

Game	ϕ	β
(7; 3, 1, 1, 1, 1, 1, 1)	(5, 2, 2, 2, 2, 2)/15	(3, 2, 2, 2, 2, 2)/13
(7; 3, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(7; 3, 2, 1, 1, 1, 1, 1)	(26, 14, 5, 5, 5, 5)/60	(6, 5, 2, 2, 2, 2)/19
(7; 3, 2, 1, 1, 1, 1)	(7, 7, 2, 2, 2)/20	(2, 2, 1, 1, 1)/7
(7; 3, 2, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 3, 2, 2, 1, 1, 1, 1)	(20, 11, 11, 6, 6, 6)/60	(15, 9, 9, 5, 5, 5)/48
(7; 3, 2, 2, 1, 1)	(26, 11, 11, 6, 6)/60	(3, 2, 2, 1, 1)/9
(7; 3, 2, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(7; 3, 2, 2, 2, 1, 1, 1)	(19, 11, 11, 11, 4, 4)/60	(8, 5, 5, 5, 2, 2)/27
(7; 3, 2, 2, 2, 1, 1)	(18, 13, 13, 13, 3)/60	(7, 5, 5, 5, 1)/23
(7; 3, 2, 2, 2, 2, 1, 1)	(8, 5, 5, 5, 5, 2)/30	(8, 5, 5, 5, 5, 2)/30
(7; 3, 2, 2, 2, 2, 2)	(8, 3, 3, 3, 3)/20	(5, 2, 2, 2, 2)/13
(7; 3, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(7; 3, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 3, 3, 1, 1, 1, 1, 1, 1)	(4, 4, 1, 1, 1, 1)/12	(5, 5, 1, 1, 1, 1)/14
(7; 3, 3, 1, 1, 1, 1)	(27, 27, 2, 2, 2)/60	(7, 7, 1, 1, 1)/17
(7; 3, 3, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(7; 3, 3, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 3, 3, 2, 1, 1, 1, 1, 1)	(17, 17, 11, 5, 5, 5)/60	(15, 15, 9, 5, 5, 5)/54
(7; 3, 3, 2, 1, 1, 1)	(9, 9, 4, 4, 4)/30	(7, 7, 3, 3, 3)/23
(7; 3, 3, 2, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(7; 3, 3, 2, 2, 1, 1, 1, 1)	(16, 16, 9, 9, 5, 5)/60	(15, 15, 9, 9, 5, 5)/58
(7; 3, 3, 2, 2, 1, 1)	(17, 17, 12, 12, 2)/60	(7, 7, 5, 5, 1)/25
(7; 3, 3, 2, 2, 2)	(3, 3, 2, 2, 2)/12	(7, 7, 5, 5, 5)/29
(7; 3, 3, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 3, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 3, 3, 3, 1, 1, 1, 1, 1, 1)	(17, 17, 17, 3, 3, 3)/60	(5, 5, 5, 1, 1, 1, 1)/18
(7; 3, 3, 3, 1, 1, 1)	(8, 8, 8, 3, 3)/30	(7, 7, 7, 3, 3)/27
(7; 3, 3, 3, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 3, 3, 3, 2, 1, 1, 1, 1)	(8, 8, 8, 3, 3)/30	(7, 7, 7, 3, 3)/27
(7; 3, 3, 3, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 3, 3, 3, 3)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 3, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(7; 4, 1, 1, 1, 1, 1, 1, 1)	(5, 1, 1, 1, 1, 1, 1)/10	(8, 3, 3, 3, 3, 3, 3)/23
(7; 4, 1, 1, 1, 1, 1, 1)	(8, 3, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3, 3)/17
(7; 4, 1, 1, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(7; 4, 2, 1, 1, 1, 1, 1, 1)	(17, 5, 2, 2, 2, 2, 2)/30	(10, 5, 2, 2, 2, 2, 2)/23
(7; 4, 2, 1, 1, 1, 1, 1)	(6, 3, 1, 1, 1, 1)/12	(4, 3, 1, 1, 1, 1)/10
(7; 4, 2, 1, 1, 1)	(5, 5, 1, 1, 1)/12	(3, 3, 1, 1, 1)/8
(7; 4, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 4, 2, 2, 1, 1, 1, 1, 1)	(27, 9, 9, 5, 5, 5, 5)/60	(11, 4, 4, 2, 2, 2, 2)/25
(7; 4, 2, 2, 1, 1, 1, 1)	(34, 9, 9, 4, 4, 4)/60	(5, 2, 2, 1, 1, 1)/11
(7; 4, 2, 2, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(7; 4, 2, 2, 2, 1, 1, 1, 1)	(25, 9, 9, 9, 4, 4, 4)/60	(11, 4, 4, 4, 2, 2, 2)/27
(7; 4, 2, 2, 2, 1, 1)	(8, 3, 3, 3, 3, 3)/20	(5, 2, 2, 2, 2, 2)/13
(7; 4, 2, 2, 2, 2)	(8, 3, 3, 3, 3, 3)/20	(5, 2, 2, 2, 2, 2)/13
(7; 4, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(7; 4, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 4, 3, 1, 1, 1, 1, 1, 1)	(26, 14, 5, 5, 5, 5, 5)/60	(5, 3, 1, 1, 1, 1, 1)/12
(7; 4, 3, 1, 1, 1, 1)	(11, 6, 1, 1, 1, 1)/20	(9, 7, 1, 1, 1, 1)/19
(7; 4, 3, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(7; 4, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 4, 3, 2, 1, 1, 1, 1, 1)	(23, 14, 11, 4, 4, 4, 4)/60	(10, 6, 5, 2, 2, 2, 2)/27
(7; 4, 3, 2, 1, 1, 1)	(5, 3, 2, 1, 1, 1)/12	(5, 3, 2, 1, 1, 1)/12
(7; 4, 3, 2, 1)	(7, 3, 1, 1, 1)/12	(5, 3, 1, 1, 1)/10
(7; 4, 3, 2, 2, 1, 1, 1, 1)	(24, 14, 9, 9, 4, 4)/60	(5, 3, 2, 2, 1, 1, 1)/13
(7; 4, 3, 2, 2)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(7; 4, 3, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 4, 3, 3, 1, 1, 1, 1, 1)	(22, 17, 17, 2, 2, 2)/60	(9, 7, 7, 1, 1, 1)/25
(7; 4, 3, 3, 1)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(7; 4, 3, 3, 2)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(7; 4, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 4, 4, 1, 1, 1, 1, 1, 1, 1)	(3, 3, 1, 1, 1, 1, 1, 1)/10	(8, 8, 3, 3, 3, 3, 3, 3)/28
(7; 4, 4, 1, 1, 1, 1, 1)	(7, 7, 2, 2, 2, 2)/20	(4, 4, 1, 1, 1, 1, 1)/11
(7; 4, 4, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(7; 4, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 4, 4, 2, 1, 1, 1, 1, 1)	(19, 19, 14, 4, 4, 4, 4)/60	(4, 4, 3, 1, 1, 1, 1)/13
(7; 4, 4, 2, 1)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(7; 4, 4, 2, 2)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(7; 4, 4, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 4, 4, 3, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(7; 4, 4, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 4, 4, 4)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(7; 5, 1, 1, 1, 1, 1)	(10, 1, 1, 1, 1, 1)/15	(13, 2, 2, 2, 2, 2)/23
(7; 5, 1, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3)/23
(7; 5, 1, 1, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(7; 5, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 5, 2, 1, 1, 1, 1)	(14, 2, 1, 1, 1, 1)/20	(27, 5, 3, 3, 3, 3)/44
(7; 5, 2, 1, 1, 1)	(39, 9, 4, 4, 4)/60	(6, 2, 1, 1, 1)/11
(7; 5, 2, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(7; 5, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 5, 2, 2, 1, 1, 1)	(34, 7, 7, 4, 4, 4)/60	(27, 5, 5, 3, 3, 3)/46
(7; 5, 2, 2, 1, 1)	(42, 7, 7, 2, 2)/60	(13, 3, 3, 1, 1)/21
(7; 5, 2, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(7; 5, 2, 2, 2, 1)	(33, 8, 8, 8, 3)/60	(13, 3, 3, 3, 1)/23
(7; 5, 2, 2, 2)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 5, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 5, 3, 1, 1, 1, 1)	(32, 8, 5, 5, 5, 5)/60	(13, 3, 2, 2, 2, 2)/24
(7; 5, 3, 1, 1, 1)	(39, 9, 4, 4, 4)/60	(6, 2, 1, 1, 1)/11
(7; 5, 3, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(7; 5, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 5, 3, 2, 1, 1)	(6, 2, 2, 1, 1)/12	(6, 2, 2, 1, 1)/12
(7; 5, 3, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(7; 5, 3, 2, 2)	(3, 1, 1, 1)/6	(3, 1, 1, 1)/6
(7; 5, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 5, 3, 3, 1)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(7; 5, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 5, 4, 1, 1, 1)	(27, 12, 7, 7, 7)/60	(11, 5, 3, 3, 3)/25
(7; 5, 4, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(7; 5, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 5, 4, 2, 1)	(5, 3, 3, 1)/12	(5, 3, 3, 1)/12
(7; 5, 4, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 5, 4, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(7; 5, 5, 1, 1)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(7; 5, 5, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(7; 5, 5, 2)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(7; 6, 1, 1, 1, 1, 1)	(25, 1, 1, 1, 1, 1)/30	(31, 1, 1, 1, 1, 1)/36
(7; 6, 1, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(7; 6, 1, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 2, 1, 1, 1, 1)	(25, 1, 1, 1, 1, 1)/30	(31, 1, 1, 1, 1, 1)/36
(7; 6, 2, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(7; 6, 2, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 2, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 2, 2, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(7; 6, 2, 2, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 2, 2, 2)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 3, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(7; 6, 3, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 3, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 3, 2, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 4, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(7; 6, 4, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 4, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(7; 6, 5, 1)	(4, 1, 1)/6	(3, 1, 1)/5

Game	ϕ	β
(8; 2, 2, 1, 1, 1)	(1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1)/6
(8; 2, 2, 2, 1, 1)	(4, 4, 4, 1, 1, 1)/15	(2, 2, 2, 1, 1, 1)/9
(8; 2, 2, 2, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(8; 2, 2, 2, 2, 1, 1)	(11, 11, 11, 11, 8, 8)/60	(3, 3, 3, 3, 2, 2)/16
(8; 2, 2, 2, 2, 1)	(1, 1, 1, 1, 0)/4	(1, 1, 1, 1, 0)/4
(8; 2, 2, 2, 2, 2, 1)	(1, 1, 1, 1, 0)/5	(1, 1, 1, 1, 0)/5
(8; 2, 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1)/6
(8; 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(8; 2, 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4

Game	ϕ	β
(8; 3, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1)/6
(8; 3, 2, 1, 1, 1, 1)	(3, 3, 1, 1, 1, 1)/10	(5, 5, 3, 3, 3, 3)/22
(8; 3, 2, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(8; 3, 2, 2, 1, 1, 1)	(23, 11, 11, 5, 5, 5)/60	(9, 7, 7, 3, 3, 3)/32
(8; 3, 2, 2, 1, 1)	(6, 6, 6, 1, 1)/20	(3, 3, 3, 1, 1)/11
(8; 3, 2, 2, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 3, 2, 2, 2, 1, 1)	(17, 11, 11, 11, 5, 5)/60	(3, 2, 2, 2, 1, 1)/11
(8; 3, 2, 2, 2, 1)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(8; 3, 2, 2, 2, 2, 1)	(6, 3, 3, 3, 3, 2)/20	(7, 4, 4, 4, 4, 3)/26
(8; 3, 2, 2, 2, 2)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(8; 3, 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 3, 3, 1, 1, 1, 1)	(8, 8, 1, 1, 1, 1)/20	(11, 11, 3, 3, 3, 3)/34
(8; 3, 3, 1, 1, 1)	(7, 7, 2, 2, 2)/20	(2, 2, 1, 1, 1)/7
(8; 3, 3, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 3, 3, 2, 1, 1, 1)	(17, 17, 8, 6, 6, 6)/60	(6, 6, 3, 2, 2, 2)/21
(8; 3, 3, 2, 1, 1)	(23, 23, 8, 3, 3)/60	(5, 5, 3, 1, 1)/15
(8; 3, 3, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(8; 3, 3, 2, 2, 1, 1)	(16, 16, 11, 11, 3, 3)/60	(13, 13, 9, 9, 3, 3)/50
(8; 3, 3, 2, 2, 1)	(16, 16, 11, 11, 6)/60	(3, 3, 2, 2, 1)/11
(8; 3, 3, 2, 2, 2)	(9, 9, 4, 4, 4)/30	(7, 7, 3, 3, 3)/23
(8; 3, 3, 2, 2)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(8; 3, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 3, 3, 3, 1, 1, 1)	(7, 7, 7, 3, 3, 3)/30	(2, 2, 2, 1, 1, 1)/9
(8; 3, 3, 3, 1, 1)	(14, 14, 14, 9, 9)/60	(5, 5, 5, 3, 3)/21
(8; 3, 3, 3, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(8; 3, 3, 3, 2, 1)	(1, 1, 1, 1, 0)/4	(1, 1, 1, 1, 0)/4
(8; 3, 3, 3, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 3, 3, 3, 3)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 3, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(8; 4, 1, 1, 1, 1, 1)	(5, 2, 2, 2, 2, 2)/15	(3, 2, 2, 2, 2, 2)/13
(8; 4, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(8; 4, 2, 1, 1, 1, 1)	(26, 14, 5, 5, 5, 5)/60	(6, 5, 2, 2, 2, 2)/19
(8; 4, 2, 1, 1, 1)	(7, 7, 2, 2, 2)/20	(2, 2, 1, 1, 1)/7
(8; 4, 2, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 4, 2, 2, 1, 1, 1)	(30, 9, 9, 4, 4, 4)/60	(4, 2, 2, 1, 1, 1)/11
(8; 4, 2, 2, 1, 1)	(26, 11, 11, 6, 6)/60	(3, 2, 2, 1, 1)/9
(8; 4, 2, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(8; 4, 2, 2, 2, 1, 1)	(23, 9, 9, 9, 5, 5)/60	(9, 4, 4, 4, 2, 2)/25
(8; 4, 2, 2, 2, 1)	(3, 1, 1, 1, 0)/6	(2, 1, 1, 1, 0)/5
(8; 4, 2, 2, 2, 2)	(8, 3, 3, 3, 3)/20	(5, 2, 2, 2, 2)/13
(8; 4, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 4, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 4, 3, 1, 1, 1, 1)	(10, 6, 1, 1, 1, 1)/20	(8, 7, 1, 1, 1, 1)/19
(8; 4, 3, 1, 1, 1)	(27, 27, 2, 2, 2)/60	(7, 7, 1, 1, 1)/17
(8; 4, 3, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(8; 4, 3, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 4, 3, 2, 1, 1, 1)	(23, 14, 8, 5, 5, 5)/60	(9, 6, 3, 2, 2, 2)/24
(8; 4, 3, 2, 1, 1)	(6, 3, 1, 1, 1)/12	(4, 3, 1, 1, 1)/10
(8; 4, 3, 2, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(8; 4, 3, 2, 2, 1)	(4, 3, 2, 2, 1)/12	(4, 3, 2, 2, 1)/12
(8; 4, 3, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 4, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 4, 3, 3, 1, 1)	(22, 12, 12, 7, 7)/60	(9, 5, 5, 3, 3)/25
(8; 4, 3, 3, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 4, 3, 3, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 4, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 4, 4, 1, 1, 1, 1)	(11, 11, 2, 2, 2, 2)/30	(8, 8, 1, 1, 1, 1)/20
(8; 4, 4, 1, 1, 1)	(1, 1, 0, 0, 0)/2	(1, 1, 0, 0, 0)/2
(8; 4, 4, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(8; 4, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 4, 4, 2, 1, 1)	(7, 7, 2, 2, 2)/20	(4, 4, 1, 1, 1)/11
(8; 4, 4, 2, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(8; 4, 4, 2, 2)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(8; 4, 4, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 4, 4, 3, 1)	(2, 2, 1, 1)/6	(2, 2, 1, 1)/6
(8; 4, 4, 3)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 4, 4, 4)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(8; 5, 1, 1, 1, 1, 1)	(5, 1, 1, 1, 1, 1)/10	(8, 3, 3, 3, 3, 3)/23
(8; 5, 1, 1, 1, 1)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(8; 5, 1, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(8; 5, 2, 1, 1, 1, 1)	(17, 5, 2, 2, 2, 2)/30	(10, 5, 2, 2, 2, 2)/23
(8; 5, 2, 1, 1, 1)	(6, 3, 1, 1, 1)/12	(4, 3, 1, 1, 1)/10
(8; 5, 2, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(8; 5, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 5, 2, 2, 1, 1, 1)	(37, 7, 7, 3, 3, 3)/60	(23, 7, 7, 3, 3, 3)/46
(8; 5, 2, 2, 1, 1)	(34, 9, 9, 4, 4)/60	(5, 2, 2, 1, 1)/11
(8; 5, 2, 2, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 5, 2, 2, 2, 1)	(6, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3)/23
(8; 5, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 5, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 5, 3, 1, 1, 1, 1)	(12, 4, 1, 1, 1, 1)/20	(21, 11, 3, 3, 3, 3)/44
(8; 5, 3, 1, 1, 1)	(11, 6, 1, 1, 1)/20	(9, 7, 1, 1, 1)/19
(8; 5, 3, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(8; 5, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 5, 3, 2, 1, 1)	(37, 12, 7, 2, 2)/60	(11, 5, 3, 1, 1, 1)/21
(8; 5, 3, 2, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 5, 3, 2, 2)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 5, 3, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 5, 3, 3, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(8; 5, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 5, 4, 1, 1, 1)	(11, 6, 1, 1, 1)/20	(9, 7, 1, 1, 1)/19
(8; 5, 4, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(8; 5, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 5, 4, 2, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 5, 4, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 5, 4, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 5, 5, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(8; 5, 5, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 5, 5, 2)	(1, 1, 0)/2	(1, 1, 0)/2

Game	ϕ	β
(8; 6, 1, 1, 1, 1)	(10, 1, 1, 1, 1)/15	(13, 2, 2, 2, 2)/23
(8; 6, 1, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3)/23
(8; 6, 1, 1, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(8; 6, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(8; 6, 2, 1, 1, 1, 1)	(14, 2, 1, 1, 1, 1)/20	(27, 5, 3, 3, 3, 3)/44
(8; 6, 2, 1, 1, 1)	(39, 9, 4, 4, 4)/60	(6, 2, 1, 1, 1)/11
(8; 6, 2, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 6, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 6, 2, 2, 1, 1)	(42, 7, 7, 2, 2)/60	(13, 3, 3, 1, 1)/21
(8; 6, 2, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(8; 6, 2, 2, 2)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(8; 6, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 6, 3, 1, 1, 1)	(39, 9, 4, 4, 4)/60	(6, 2, 1, 1, 1)/11
(8; 6, 3, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 6, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 6, 3, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(8; 6, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 6, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 6, 4, 1, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(8; 6, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(8; 6, 4, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 6, 5, 1)	(1, 1, 0)/2	(1, 1, 0)/2

Game	ϕ	β
(8; 7, 1, 1, 1, 1, 1)	(25, 1, 1, 1, 1, 1)/30	(31, 1, 1, 1, 1, 1)/36
(8; 7, 1, 1, 1, 1)	(16, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1)/19
(8; 7, 1, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(8; 7, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 7, 2, 1, 1, 1, 1)	(16, 1, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1, 1)/19
(8; 7, 2, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(8; 7, 2, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 7, 2, 2, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(8; 7, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 7, 3, 1, 1, 1)	(9, 1, 1, 1, 1)/12	(7, 1, 1, 1, 1)/10
(8; 7, 3, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 7, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(8; 7, 4, 1)	(4, 1, 1)/6	(3, 1, 1)/5

Game	ϕ	β
(9; 2, 2, 2, 1, 1, 1, 1)	(1, 1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1, 1)/6
(9; 2, 2, 2, 2, 1, 1, 1)	(7, 7, 7, 7, 1, 1, 1)/30	(3, 3, 3, 3, 1, 1, 1)/14
(9; 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1, 1)/5
(9; 2, 2, 2, 2, 2, 1)	(1, 1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1, 1)/6
(9; 2, 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1, 1)/6
(9; 2, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1, 1)/5
(9; 3, 2, 1, 1, 1, 1, 1)	(1, 1, 1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1, 1, 1)/6

Game	ϕ	β
(9; 3, 2, 2, 1, 1, 1, 1)	(4, 4, 4, 1, 1, 1, 1)/15	(2, 2, 2, 1, 1, 1, 1)/9
(9; 3, 2, 2, 1, 1)	(1, 1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1, 1)/5
(9; 3, 2, 2, 2, 1, 1, 1)	(7, 3, 3, 3, 2, 2, 2)/20	(7, 5, 5, 5, 3, 3, 3)/28
(9; 3, 2, 2, 2, 1)	(1, 1, 1, 1, 1, 0)/4	(1, 1, 1, 1, 1, 0)/4
(9; 3, 2, 2, 2, 2, 1)	(14, 11, 11, 11, 11, 2)/60	(9, 7, 7, 7, 7, 1)/38
(9; 3, 2, 2, 2, 2, 2)	(8, 3, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3, 3)/17
(9; 3, 2, 2, 2, 2)	(1, 1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1, 1)/4
(9; 3, 3, 1, 1, 1, 1, 1)	(3, 3, 1, 1, 1, 1, 1)/10	(5, 5, 3, 3, 3, 3, 3)/22
(9; 3, 3, 1, 1, 1)	(1, 1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1, 1)/5
(9; 3, 3, 2, 1, 1, 1, 1)	(7, 7, 3, 1, 1, 1, 1)/20	(4, 4, 3, 1, 1, 1, 1)/14
(9; 3, 3, 2, 1, 1)	(6, 6, 6, 1, 1, 1)/20	(3, 3, 3, 1, 1, 1, 1)/11
(9; 3, 3, 2, 1)	(1, 1, 1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1, 1, 1)/4
(9; 3, 3, 2, 2, 1, 1)	(5, 5, 3, 3, 2, 2, 2)/20	(5, 5, 3, 3, 2, 2, 2)/20
(9; 3, 3, 2, 2, 1)	(7, 7, 2, 2, 2, 2)/20	(2, 2, 1, 1, 1, 1, 1)/7
(9; 3, 3, 2, 2, 2)	(1, 1, 1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1, 1, 1)/5
(9; 3, 3, 2, 2)	(1, 1, 1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1, 1, 1)/4
(9; 3, 3, 3, 1, 1, 1, 1)	(7, 7, 7, 3, 3, 3, 3)/30	(3, 3, 3, 1, 1, 1, 1, 1)/12
(9; 3, 3, 3, 1, 1)	(1, 1, 1, 1, 0, 0)/3	(1, 1, 1, 1, 0, 0)/3
(9; 3, 3, 3, 1)	(1, 1, 1, 1, 0, 0)/3	(1, 1, 1, 1, 0, 0)/3
(9; 3, 3, 3, 2, 1)	(14, 14, 14, 9, 9)/60	(5, 5, 5, 3, 3, 3)/21
(9; 3, 3, 3, 2)	(1, 1, 1, 1, 0, 0)/3	(1, 1, 1, 1, 0, 0)/3
(9; 3, 3, 3, 3)	(1, 1, 1, 1, 1, 1, 1)/4	(1, 1, 1, 1, 1, 1, 1)/4
(9; 3, 3, 3, 3)	(1, 1, 1, 1, 1, 1, 1)/3	(1, 1, 1, 1, 1, 1, 1)/3

Game	ϕ	β
(9; 4, 1, 1, 1, 1)	(1, 1, 1, 1, 1)/6	(1, 1, 1, 1, 1)/6
(9; 4, 2, 1, 1, 1)	(3, 3, 1, 1, 1)/10	(5, 5, 3, 3, 3)/22
(9; 4, 2, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(9; 4, 2, 2, 1, 1)	(23, 11, 11, 5, 5, 5)/60	(9, 7, 7, 3, 3, 3)/32
(9; 4, 2, 2, 1, 1)	(6, 6, 6, 1, 1)/20	(3, 3, 3, 1, 1)/11
(9; 4, 2, 2, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(9; 4, 2, 2, 2, 1, 1)	(9, 3, 3, 3, 1, 1)/20	(13, 7, 7, 7, 3, 3)/40
(9; 4, 2, 2, 2, 1)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(9; 4, 2, 2, 2, 2)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(9; 4, 2, 2, 2)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(9; 4, 3, 1, 1, 1, 1)	(8, 8, 1, 1, 1, 1)/20	(11, 11, 3, 3, 3, 3)/34
(9; 4, 3, 1, 1, 1)	(7, 7, 2, 2, 2)/20	(2, 2, 1, 1, 1)/7
(9; 4, 3, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(9; 4, 3, 2, 1, 1, 1)	(27, 15, 6, 4, 4, 4)/60	(13, 11, 5, 3, 3, 3)/38
(9; 4, 3, 2, 1, 1)	(23, 23, 8, 3, 3)/60	(5, 5, 3, 1, 1)/15
(9; 4, 3, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(9; 4, 3, 2, 2, 1)	(28, 13, 8, 8, 3)/60	(7, 5, 3, 3, 1)/19
(9; 4, 3, 2, 2)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 4, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 4, 3, 3, 1, 1)	(26, 11, 11, 6, 6)/60	(3, 2, 2, 1, 1)/9
(9; 4, 3, 3, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(9; 4, 3, 3, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 4, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 4, 4, 1, 1, 1, 1)	(28, 28, 1, 1, 1, 1)/60	(15, 15, 1, 1, 1, 1)/34
(9; 4, 4, 1, 1, 1)	(27, 27, 2, 2, 2)/60	(7, 7, 1, 1, 1)/17
(9; 4, 4, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 4, 4, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 4, 4, 2, 1, 1)	(27, 27, 2, 2, 2)/60	(7, 7, 1, 1, 1)/17
(9; 4, 4, 2, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 4, 4, 2, 2)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 4, 4, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 4, 4, 3, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 4, 4, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 4, 4, 4)	(1, 1, 1)/3	(1, 1, 1)/3

Game	ϕ	β
(9; 5, 1, 1, 1, 1, 1)	(5, 2, 2, 2, 2, 2)/15	(3, 2, 2, 2, 2, 2)/13
(9; 5, 1, 1, 1, 1)	(1, 1, 1, 1, 1)/5	(1, 1, 1, 1, 1)/5
(9; 5, 2, 1, 1, 1, 1)	(26, 14, 5, 5, 5, 5)/60	(6, 5, 2, 2, 2, 2)/19
(9; 5, 2, 1, 1, 1)	(7, 7, 2, 2, 2)/20	(2, 2, 1, 1, 1)/7
(9; 5, 2, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(9; 5, 2, 2, 1, 1, 1)	(30, 9, 9, 4, 4, 4)/60	(4, 2, 2, 1, 1, 1)/11
(9; 5, 2, 2, 1, 1)	(26, 11, 11, 6, 6)/60	(3, 2, 2, 1, 1)/9
(9; 5, 2, 2, 1)	(1, 1, 1, 0)/3	(1, 1, 1, 0)/3
(9; 5, 2, 2, 2, 1)	(3, 1, 1, 1, 0)/6	(2, 1, 1, 1, 0)/5
(9; 5, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 5, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 5, 3, 1, 1, 1, 1)	(10, 6, 1, 1, 1, 1)/20	(8, 7, 1, 1, 1, 1)/19
(9; 5, 3, 1, 1, 1)	(27, 27, 2, 2, 2)/60	(7, 7, 1, 1, 1)/17
(9; 5, 3, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 5, 3, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 5, 3, 2, 1, 1)	(6, 3, 1, 1, 1)/12	(4, 3, 1, 1, 1)/10
(9; 5, 3, 2, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 5, 3, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 5, 3, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 5, 3, 3, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 5, 3, 3)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 5, 4, 1, 1, 1)	(1, 1, 0, 0, 0)/2	(1, 1, 0, 0, 0)/2
(9; 5, 4, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(9; 5, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 5, 4, 2, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(9; 5, 4, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 5, 4, 3)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 5, 5, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(9; 5, 5, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 5, 5, 2)	(1, 1, 0)/2	(1, 1, 0)/2

Game	ϕ	β
(9; 6, 1, 1, 1, 1, 1, 1)	(5, 1, 1, 1, 1, 1, 1)/10	(8, 3, 3, 3, 3, 3, 3)/23
(9; 6, 1, 1, 1, 1, 1)	(8, 3, 3, 3, 3)/20	(5, 3, 3, 3, 3)/17
(9; 6, 1, 1, 1)	(1, 1, 1, 1)/4	(1, 1, 1, 1)/4
(9; 6, 2, 1, 1, 1, 1, 1)	(17, 5, 2, 2, 2, 2, 2)/30	(10, 5, 2, 2, 2, 2, 2)/23
(9; 6, 2, 1, 1, 1, 1)	(6, 3, 1, 1, 1)/12	(4, 3, 1, 1, 1)/10
(9; 6, 2, 1, 1)	(5, 5, 1, 1)/12	(3, 3, 1, 1)/8
(9; 6, 2, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 6, 2, 2, 1, 1, 1)	(34, 9, 9, 4, 4)/60	(5, 2, 2, 1, 1)/11
(9; 6, 2, 2, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 6, 2, 2, 2)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 6, 2, 2)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 6, 3, 1, 1, 1, 1)	(11, 6, 1, 1, 1, 1)/20	(9, 7, 1, 1, 1, 1)/19
(9; 6, 3, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(9; 6, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 6, 3, 2, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(9; 6, 3, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 6, 3, 3)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 6, 4, 1, 1)	(1, 1, 0, 0)/2	(1, 1, 0, 0)/2
(9; 6, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 6, 4, 2)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 6, 5, 1)	(1, 1, 0)/2	(1, 1, 0)/2

Game	ϕ	β
(9; 7, 1, 1, 1, 1, 1, 1)	(10, 1, 1, 1, 1, 1, 1)/15	(13, 2, 2, 2, 2, 2, 2)/23
(9; 7, 1, 1, 1, 1, 1)	(6, 1, 1, 1, 1)/10	(11, 3, 3, 3, 3, 3)/23
(9; 7, 1, 1, 1)	(3, 1, 1, 1)/6	(2, 1, 1, 1)/5
(9; 7, 1, 1)	(1, 1, 1)/3	(1, 1, 1)/3
(9; 7, 2, 1, 1, 1, 1)	(39, 9, 4, 4, 4)/60	(6, 2, 1, 1, 1, 1)/11
(9; 7, 2, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(9; 7, 2, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 7, 2, 2, 1)	(4, 1, 1, 0)/6	(3, 1, 1, 0)/5
(9; 7, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 7, 3, 1, 1)	(7, 3, 1, 1)/12	(5, 3, 1, 1)/10
(9; 7, 3, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 7, 3, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 7, 4, 1)	(1, 1, 0)/2	(1, 1, 0)/2
(9; 8, 1, 1, 1, 1, 1)	(16, 1, 1, 1, 1, 1)/20	(15, 1, 1, 1, 1, 1)/19
(9; 8, 1, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(9; 8, 1, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 8, 2, 1, 1)	(9, 1, 1, 1)/12	(7, 1, 1, 1)/10
(9; 8, 2, 1)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 8, 2, 2)	(4, 1, 1)/6	(3, 1, 1)/5
(9; 8, 3, 1)	(4, 1, 1)/6	(3, 1, 1)/5

2 THE CYCLIC RULE

RELATED TO [Section 5.1.6.1](#), page 389:

When $\{x, y, z\}$ are functionally related, then the relationship between the partial derivatives (holding the third variable constant) is

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

EXAMPLE If $x^2 + y^3z = 5$ then

$$\begin{aligned} \left(\frac{\partial}{\partial y}\right)_z (x^2 + y^3z = 5) &\implies 2x \left(\frac{\partial x}{\partial y}\right)_z + 3y^2z = 0 && \text{or} && \left(\frac{\partial x}{\partial y}\right)_z = -\frac{3y^2z}{2x} \\ \left(\frac{\partial}{\partial z}\right)_x (x^2 + y^3z = 5) &\implies 3y^2 \left(\frac{\partial y}{\partial z}\right)_x z + y^3 = 0 && \text{or} && \left(\frac{\partial y}{\partial z}\right)_x = -\frac{y}{3z}, \\ \left(\frac{\partial}{\partial x}\right)_y (x^2 + y^3z = 5) &\implies 2x + y^3 \left(\frac{\partial z}{\partial x}\right)_y = 0 && \text{or} && \left(\frac{\partial z}{\partial x}\right)_y = -\frac{2x}{y^3} \end{aligned}$$